Prof. Dr. Roderich Tumulka
Mathematisches Institut, Universität Tübingen

Assignment 8
Wintersemester 2017/18

Foundations of Quantum Mechanics

Written homework due Wednesday January 17, 2018

Exercise 31: Essay question. Describe the Einstein–Podolsky–Rosen argument (either in terms of position and momentum or in terms of spin matrices).

Exercise 32: Consider two random variables X, Y that assume only values ± 1. Their joint distribution can be described by a 2×2 table of probabilities. **(a)** Give a typical example of such a table, but not the one of (16.12) for any θ. For your table, compute **(b)** the marginal distribution of X and **(c)** that of Y, as well as **(d)** the conditional distribution of X, given that $Y = +1$, **(e)** the expectation value $\mathbb{E}(X)$, and **(f)** $\mathbb{E}(XY)$.

Exercise 33: Quantum Zeno effect
Zeno of Elea (c. 490–c. 430 BCE) was a Greek philosopher who claimed that motion and time cannot exist because they are inherently paradoxical notions, a claim which he tried to support by formulating various paradoxes, including one involving Achilles and a turtle. In modern times, Alan Turing (of computer science fame, lived 1912–1954) reportedly first discovered the following effect, which was later named after Zeno because of its paradoxical flavor: Suppose a quantum particle moves in 1d, and its initial wave function $\psi_0(x)$ is concentrated in the negative half axis $(-\infty, 0)$. We want to model, as a kind of time measurement, a detector, located at the origin, that clicks when the particle arrives. To this end, we imagine that the detector performs, at times $n\tau$ with $n \in \mathbb{N}$ and time resolution $\tau > 0$, a quantum measurement of $1_{x \geq 0}$, i.e., of whether the particle is in the right half axis. The ideal detector would seem to correspond to the limit $\tau \to 0$; however, in this limit, the probability that the detector *ever* clicks is 0. "A watched pot never boils," wrote Misra und Sudarshan.[1]

Prove the following simplified version: In a 2d Hilbert space $\mathbb{C}^2$, let $\psi_0 = (1, 0)$ evolve with Hamiltonian $H = \sigma_1$, interrupted by a quantum measurement of σ_3 at times $n\tau$ for all $n \in \mathbb{N}$. For any fixed $T > 0$, the probability that any of the $\approx T/\tau$ measurements in the time interval $[0, T]$ yields the result -1 tends to 0 as $\tau \to 0$.

Exercise 34: Let $\mathscr{H}$ be a Hilbert space of finite dimension. For the purposes of this problem, a *projection-valued measure* (PVM) acting on $\mathscr{H}$ is a collection of finitely many projections P_z such that $\sum_z P_z = I$. Let $\mathscr{K}_z$ denote the subspace to which P_z projects. Show that $\mathscr{H}$ is the orthogonal sum of the $\mathscr{K}_z$, i.e., that $\mathscr{K}_z \perp \mathscr{K}_{z'}$ for $z \neq z'$ and $\mathrm{span}(\cup_z \mathscr{K}_z) = \mathscr{H}$.

Reading assignment due Friday January 19, 2018:
N. Bohr: Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Physical Review* **48**: 696–702 (1935)

[1] B. Misra and E.C.G. Sudarshan: The Zeno's paradox in quantum theory. *Journal of Mathematical Physics* **18**: 756–763 (1977)