Foundations of Quantum Mechanics

Written homework due Wednesday January 31, 2018

Exercise 35: Essay question. Describe Einstein's boxes argument.

Exercise 36: Main theorem about POVMs

The proof of the main theorem from Bohmian mechanics assumes that at the initial time t_i of the experiment, the joint wave function factorizes, $\Psi_{t_i} = \psi \otimes \phi$. What if factorization is not exactly satisfied, but only approximately? Then the probability distribution of the outcome Z is still approximately given by $\langle\psi|E(\cdot)|\psi\rangle$. To make this statement precise, suppose that

$$\Psi_t = c\psi \otimes \phi + \Delta\Psi \,, \tag{1}$$

where $\|\Delta\Psi\| \ll 1$, $\|\psi\| = \|\phi\| = 1$, and $c = \sqrt{1 - \|\Delta\Psi\|^2}$ (which is close to 1). Use the Cauchy-Schwarz inequality to show that, for any $B \subseteq \mathscr{Z}$,

$$\Big|\mathbb{P}(Z \in B) - \langle\psi|E(B)|\psi\rangle\Big| < 3\|\Delta\Psi\| \,. \tag{2}$$

Exercise 37: Tensor product

Suppose the Hilbert spaces $\mathscr{H}_a$ and $\mathscr{H}_b$ have finite dimensions. Show that for any operators $T_a : \mathscr{H}_a \to \mathscr{H}_a$ and $T_b : \mathscr{H}_b \to \mathscr{H}_b$, there is a unique operator $T_a \otimes T_b : \mathscr{H}_a \otimes \mathscr{H}_b \to \mathscr{H}_a \otimes \mathscr{H}_b$ satisfying

$$(T_a \otimes T_b)(\psi_a \otimes \psi_b) = (T_a\psi_a) \otimes (T_b\psi_b) \tag{3}$$

for all $\psi_a \in \mathscr{H}_a$ and $\psi_b \in \mathscr{H}_b$, and that it has the following properties. (*Hint*: Choose ONBs.)

(i) $(T_a \otimes T_b)^\dagger = T_a^\dagger \otimes T_b^\dagger$

(ii) $(T_a \otimes T_b)(S_a \otimes S_b) = (T_aS_a) \otimes (T_bS_b)$

(iii) $\operatorname{tr}(T_a \otimes T_b) = (\operatorname{tr} T_a)(\operatorname{tr} T_b)$.

Remark: For two non-interacting systems, the Hamiltonian is of the form $H = H_a \otimes I_b + I_a \otimes H_b$, and the time evolution is $e^{-iHt} = e^{-iH_at} \otimes e^{-iH_bt}$, i.e., of the form $U_t = U_{a,t} \otimes U_{b,t}$.

Exercise 38: Main theorem about POVMs and tensor product

Prove that in Bohmian mechanics, an experiment in which the apparatus interacts only with system a but not with system b has a POVM of the form

$$E(B) = E_a(B) \otimes I_b \tag{4}$$

for all $B \subseteq \mathscr{Z}$. To this end, adapt the proof of the main theorem of POVMs. Suppose the experiment $\mathscr{E}$ begins at time t_i and ends at time t_f, and suppose the wave function of the apparatus, system a, and system b at time t_i is $\Psi(t_i) = \phi \otimes \psi$ with $\psi \in \mathscr{H}_a \otimes \mathscr{H}_b$, so $\Psi(t_i) \in \mathscr{H}_{\text{app}} \otimes \mathscr{H}_a \otimes \mathscr{H}_b$. Use the remark at the end of Exercise 37. Assume further that the outcome Z is a function ζ of the configuration Q_{app} of the apparatus at time t_f.

Reading assignment due Friday February 2, 2018:

J. Bell: Against 'measurement.' *Physics World*, August 1990, pages 33–40.