Mini Course on Interpretations of Quantum Mechanics Lecture 1

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The slides of this talk will be available at my webpage
`http://www.math.uni-tuebingen.de/de/forschung/maphy/personen/roderichtumulka/tumulka-talks`

Recommended further reading:

- My lecture notes: `http://www.math.uni-tuebingen.de/de/forschung/maphy/lehre/ws-2019-20/quantmech/home`
- J. Bricmont: *Making Sense of Quantum Mechanics*. Springer (2016)
- T. Norsen: *Foundations of Quantum Mechanics*. Springer (2017)
- D. Dürr and D. Lazarovici: *Understanding Quantum Mechanics*. Springer (2020)

Some topics of this course

- Copenhagen interpretation
- Many worlds
- Bohmian mechanics
- Collapse theories such as GRW
- Quantum measurement and the measurement problem
- No-hidden-variables theorems
- The Einstein-Podolsky-Rosen argument
- Einsteins boxes argument
- Bells theorem and nonlocality
- Relativistic versions of Bohm/GRW/etc.

Waves and particles

A particular physical theory

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$$m_k \frac{d^2 \boldsymbol{Q}_k}{dt^2} = \sum_{j \neq k} G m_j m_k \frac{\boldsymbol{Q}_j - \boldsymbol{Q}_k}{|\boldsymbol{Q}_j - \boldsymbol{Q}_k|^3} - \sum_{j \neq k} e_j e_k \frac{\boldsymbol{Q}_j - \boldsymbol{Q}_k}{|\boldsymbol{Q}_j - \boldsymbol{Q}_k|^3} ,$$

where $m_k > 0, e_k \in \mathbb{R}, G > 0$ are constants.

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  where $m_k > 0$, $e_k \in \mathbb{R}$, $G > 0$ are constants.
- This theory is called Newtonian mechanics,
  the first sum the "gravitational force,"
  the second the "Coulomb force,"
  m_k the mass of particle k, and e_k its charge.

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$$i\hbar \frac{\partial \psi}{\partial t}(\boldsymbol{q}_1 ... \boldsymbol{q}_N, t) = -\sum_{k=1}^{N} \tfrac{\hbar^2}{2m_k} \nabla^2_{\boldsymbol{q}_k} \psi + \sum_{j \neq k} \frac{e_j e_k}{|\boldsymbol{q}_j - \boldsymbol{q}_k|} \psi\,,$$

with $\hbar > 0$ a constant.

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Bohm's equation of motion

$$\frac{d\boldsymbol{Q}_k}{dt} = \frac{\hbar}{m_k} \mathrm{Im} \frac{\nabla_{\boldsymbol{q}_k} \psi(\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N, t)}{\psi(\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N, t)} \Bigg|_{\boldsymbol{q}_j = \boldsymbol{Q}_j(t) \forall j} \quad (1)$$

can be rewritten as

$$\frac{d\boldsymbol{Q}_k}{dt} = \frac{\text{current}}{\text{density}} = \frac{\boldsymbol{j}_k(\boldsymbol{Q}_1 \ldots \boldsymbol{Q}_N)}{\rho(\boldsymbol{Q}_1 \ldots \boldsymbol{Q}_N)}$$

with prob. current $\boldsymbol{j}_k = \frac{\hbar}{m_k} \mathrm{Im}[\psi^* \nabla_k \psi]$ and prob. density $\rho = \psi^* \psi$.

Historical curiosity

Bohm (1952) wrote the eq. of motion (1) as a 2nd-order eq. for $d^2\boldsymbol{Q}_k/dt^2$ (by taking d/dt of (1)) and demanded (1) as a constraint condition on the velocity—a convoluted way of defining the same trajectories.

One more axiom of Bohmian mechanics

We write $Q(t) := (\boldsymbol{Q}_1(t), \ldots, \boldsymbol{Q}_N(t)) =:$ configuration at time t

Axiom

At the initial time $t = 0$ of the universe, $Q(0)$ is random with probability density $\rho(Q(0) = q) = |\psi(q, t = 0)|^2$. In short, $Q(0) \sim |\psi_0|^2$.

In particular, assume $\psi_0 := \psi(\cdot, t = 0) \in L^2(\mathbb{R}^{3N}, \mathbb{C})$ with $\|\psi_0\|^2 = 1$.

Equivariance theorem

If $Q(t_0) \sim |\psi_{t_0}|^2$ for one t_0, then $Q(t) \sim |\psi_t|^2$ for all t.

Sketch of proof: Prob. ρ gets transported by motion with velocity $\boldsymbol{v}_k$ according to

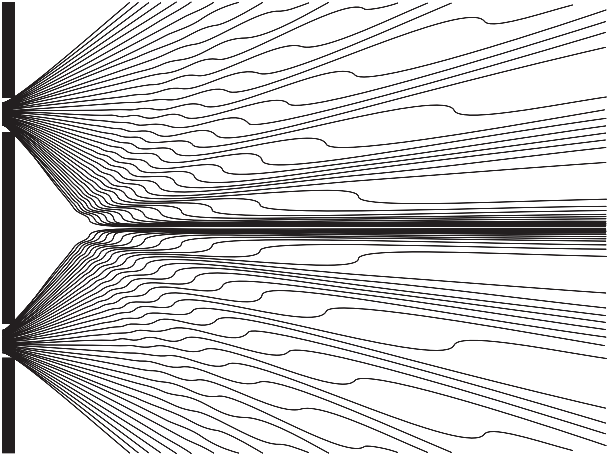
$$\frac{\partial \rho}{\partial t} = -\sum_{k=1}^{N} \nabla_k \cdot (\rho \boldsymbol{v}_k)\,.$$

The Schrödinger eq. implies that

$$\frac{\partial |\psi|^2}{\partial t} = -\sum_{k=1}^{N} \nabla_k \cdot \boldsymbol{j}_k\,.$$

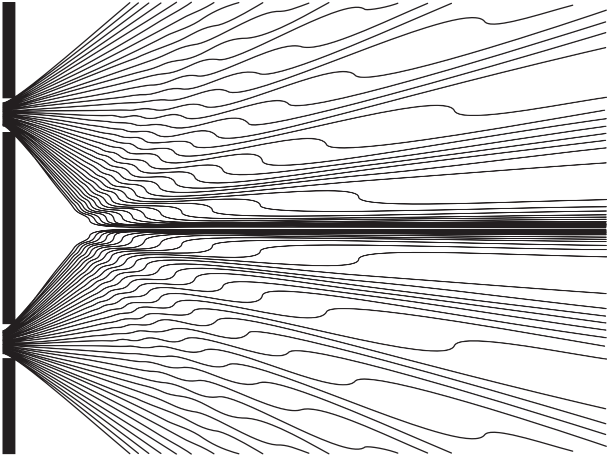
Since $\boldsymbol{v}_k = \boldsymbol{j}_k/|\psi|^2$, if $\rho = |\psi|^2$ then $\partial\rho/\partial t = \partial|\psi|^2/\partial t$.

Example: the double-slit experiment



Drawn by G. Bauer after Philippidis et al.

Shown: A double-slit and 80 possible paths of Bohm's particle. The wave passes through both slits, the particle through only one.



Most paths arrive where $|\psi|^2$ is large—that's how the interference pattern arises. If one slit gets closed, the wave passes through only one slit, which leads to different trajectories and less interference.
Bohmian mechanics takes wave–particle dualism literally: there is a wave, and there is a particle. The path of the particle depends on the wave.

Feynman Lectures on Physics (1964), Vol. 3

"We cannot make the mystery go away by "explaining" how it works." (p. 1)

"Many ideas have been concocted to try to explain the curve for P_{12} [...] None of them has succeeded." (p. 6)

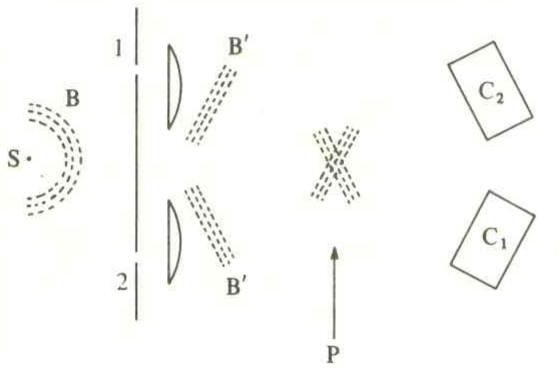
"No one has found any machinery behind the law. No one can "explain" any more than we have just "explained." No one will give you any deeper representation of the situation. We have no idea about a more basic mechanism from which these results can be deduced." (p. 10)

Richard Feynman (1918–1988)

These statements are too strong. Bohmian mechanics does just that.

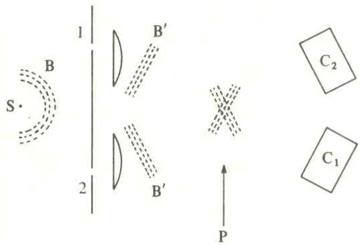
You will sometimes find inaccurate information about quantum foundations in the literature.

Wheeler's delayed-choice experiment (1978)



Wave B from source S falls on screen with slits 1 and 2. The transmitted waves B' are focussed (by lenses or potentials) into intersecting plane wave trains which fall on particle counters C_1 and C_2, unless a photo plate P is inserted in the intersection region. The experimenter can make the choice, whether or not to push in P, after the waves have passed the slits. (Drawing: J. Bell)

Delayed-choice experiment

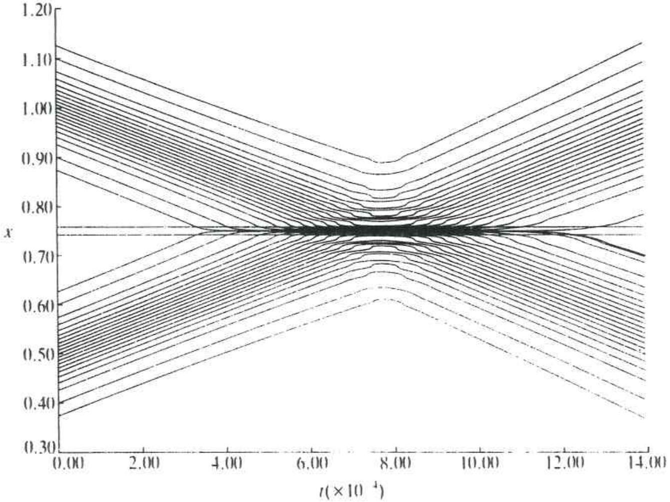


Wheeler reasoned:

With P, we see an interference pattern, so the electron must be a wave and have passed through both slits. Without P, if the lower counter C_1 clicks, then the electron must have passed through the upper slit 1 only and be a particle. Without P, if the upper counter C_2 clicks, then the electron must have passed through the lower slit 2 only and be a particle.

As if we could "choose, later, whether the [electron], earlier, went through one slit or two!" (Bell)

Delayed-choice experiment in Bohmian mechanics



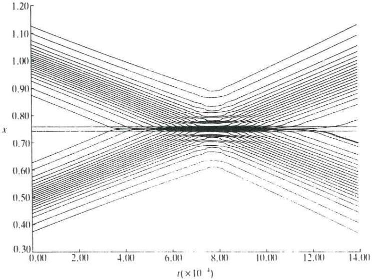
No retrocausation, no mystery. (Picture: Dewdney 1985)

Delayed-choice experiment

Now some steps in Wheeler's reasoning appear strange:

If one assumes, as in orthodox QM, that there are no trajectories, and if there were no detectors at the slits, then there is no fact about which slit the electron went through. Wheeler claims the counter reveals which slit the electron took. But how can anything reveal which slit the electron took if the electron didn't take a slit?

Delayed-choice experiment in Bohmian mechanics



One more observation (Bell 1980): If the Bohmian particle went through the upper slit, it ends up in the upper counter—the opposite of what Wheeler took for granted!

"Wheeler's fallacy"

If the lower slit is closed and only the upper slit is open, then only the lower counter clicks; and vice versa. Wheeler concluded that also when both slits are open, an electron detected in the lower counter must have passed through the upper slit—a *non sequitur*.

Rules of quantum mechanics

Rules of QM

Nearly all views about QM agree about the rules for making empirical predictions:

- Unitary evolution: The wave function ψ of an isolated system evolves according to the Schrödinger equation, $\psi_t = e^{-iHt/\hbar}\psi_0$ with H the Hamiltonian operator in Hilbert space $\mathscr{H}$.
- Born's rule: When an observer makes a "quantum measurement" of the observable $\mathscr{A}$ associated with the self-adjoint operator A with spectral decomposition $A = \sum_\alpha \alpha P_\alpha$ on a system with wave function ψ, the outcome is the eigenvalue α with probability $\|P_\alpha \psi\|^2 = \langle \psi | P_\alpha \psi \rangle$.
- Collapse rule: After a quantum measurement of $\mathscr{A}$ with outcome α, the wave function gets replaced by

$$\psi_{t+} = \frac{P_\alpha \psi_{t-}}{\|P_\alpha \psi_{t-}\|}.$$

Collapse of the wave function in Bohmian mechanics

The wave function Ψ of the universe does not collapse (but evolves according to the Schrödinger equation).
The wave function ψ of a system is the *conditional wave function*

$$\psi(x) = \mathcal{N} \, \Psi(x, Y)$$

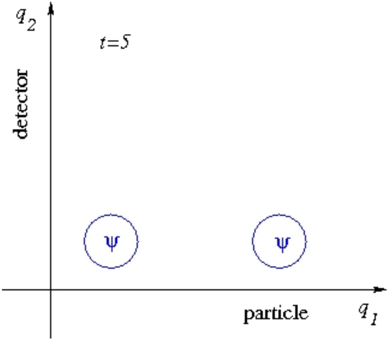
with $\mathcal{N}$ = normalizing constant, x = configuration variable of the system, Y = actual (Bohmian configuration) of the environment.

If x-system and y-system are disentangled, $\Psi(x, y) = \phi(x)\chi(y)$, and don't interact, then the conditional wave function ψ obeys its own Schrödinger eq., but in general it doesn't.

In BM, ψ collapses.

Here is why:

Evolution of Ψ in configuration space of particle + detector:



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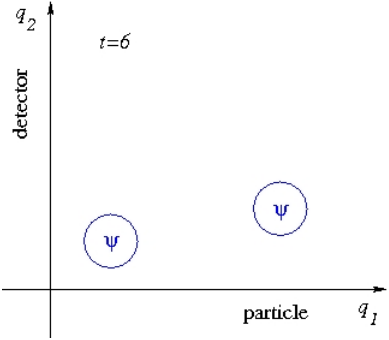

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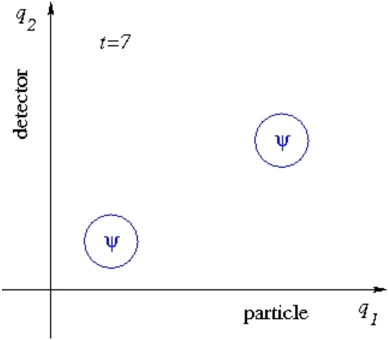

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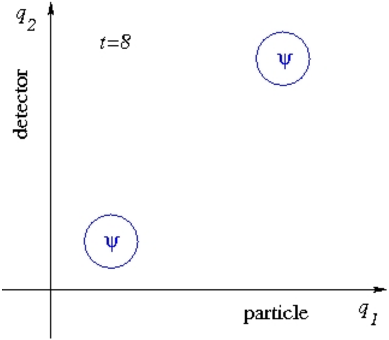


Collapse of ψ in Bohmian mechanics

- Since $Q = (X, Y) \sim |\Psi|^2$, Q lies in one of the packets; say, in the upper.
- Conditional on the configuration Y of the detector, $\psi(x)$ is a cross-section of the upper packet. That is, ψ has collapsed.
- Moreover, decoherence occurs: The two packets of Ψ do not overlap in configuration space and will not overlap any more in the future (for the next 10^{100} years). (As usual with macroscopically different packets.)
- As a consequence, $Q = (X, Y)$ will be guided only by the packet of Ψ containing Q (for the next 10^{100} years).
- Thus, ψ will follow the upper packet for the next 10^{100} years.

Measurement process more generally

Consider an ideal quantum measurement of the observable $A = \sum_\alpha \alpha P_\alpha$ with eigenvalues α and P_α the projection to the corresponding eigenspace. It begins at t_0 and ends at t_1. At t_0, the wave fct of object and apparatus is

$$\Psi(t_0) = \psi(t_0) \otimes \phi$$

with $\psi(t_0) =$ wave fct of the object, $\phi =$ ready state of the apparatus. By the Schrödinger eq., Ψ evolves to

$$\Psi(t_1) = e^{-iH(t_1-t_0)}\Psi(t_0)\,.$$

Measurement process, continued

We have that $\Psi(t_0) = \psi(t_0) \otimes \phi$ and $\Psi(t_1) = e^{-iH(t_1-t_0)}\Psi(t_0)$.

Suppose first that the object is in an eigenstate ψ_α of A. Then

$$\Psi_\alpha := \Psi(t_1) = e^{-iH(t_1-t_0)}[\psi_\alpha \otimes \phi]$$

should be a state in which the apparatus displays the value α (e.g., by the position of a needle).

Suppose next that $\psi(t_0) = \sum_\alpha c_\alpha \psi_\alpha$ is an arbitrary superposition. Then

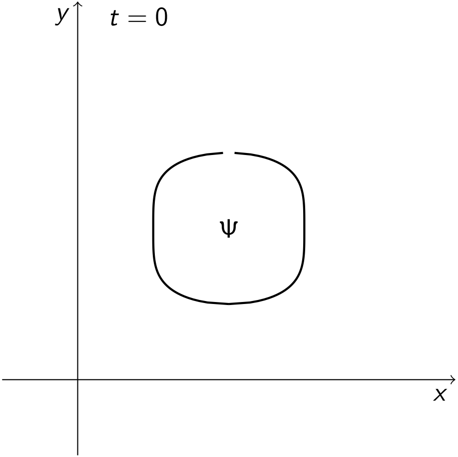
$$\Psi(t_0) = \sum_\alpha c_\alpha \, [\psi_\alpha \otimes \phi]$$

and, by linearity of the Schrödinger eq.,

$$\Psi(t_1) = \sum_\alpha c_\alpha \Psi_\alpha \,,$$

i.e., a superposition of wave functions of apparatuses displaying different outcomes.

Evolution of Ψ in configuration space of system x + apparatus y:



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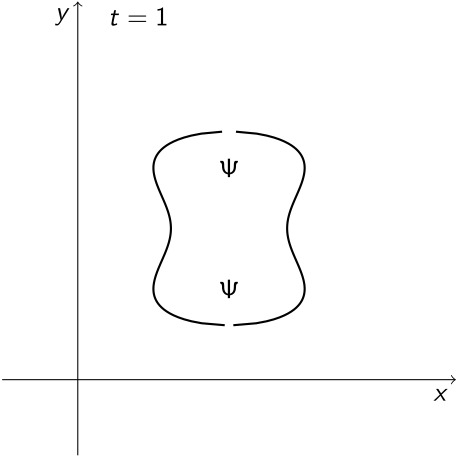

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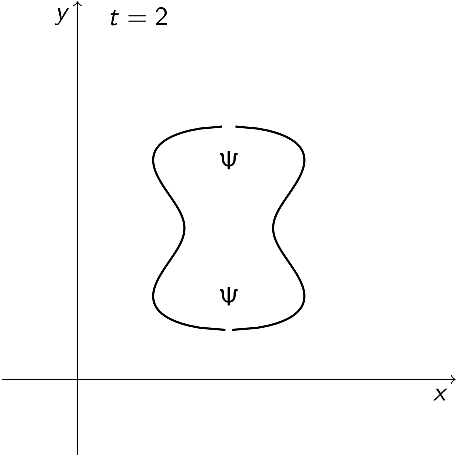

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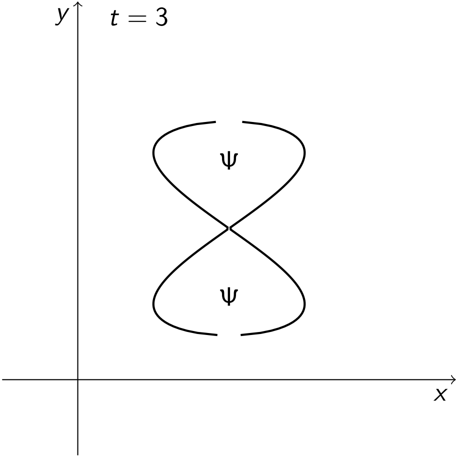

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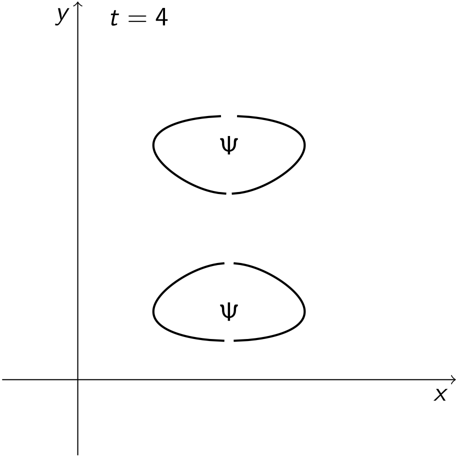

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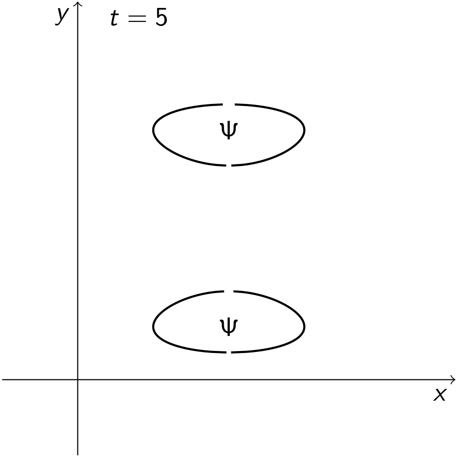

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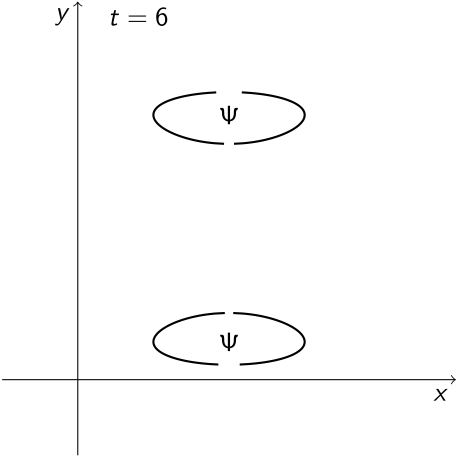


Measurement outcomes in BM

- Y provides the actual position of the needle, and thus the actual outcome $Z = f(Y)$.
- $\text{Prob}(Z = \alpha) = \|\Psi_\alpha\|^2 = |c_\alpha|^2$, in agreement with the rules of QM.
- If $\Psi_\alpha = \psi_\alpha \otimes \phi_\alpha$ for all α (i.e., if the measurement process doesn't change the state of the object), then the cond. wf is $\psi = \psi_\alpha|_{\alpha=Z}$ (collapse to eigenfunction), in agreement with the rules of QM.
- Moreover, by decoherence, also in Ψ the lower packet can henceforth be ignored.

As a consequence

Observers inhabiting a Bohmian universe (made out of Bohmian particles) observe random-looking outcomes of their experiments whose statistics agree with the rules of quantum mechanics for making predictions.

In short, Bohmian mechanics is empirically adequate.

A theory like this was believed to be impossible

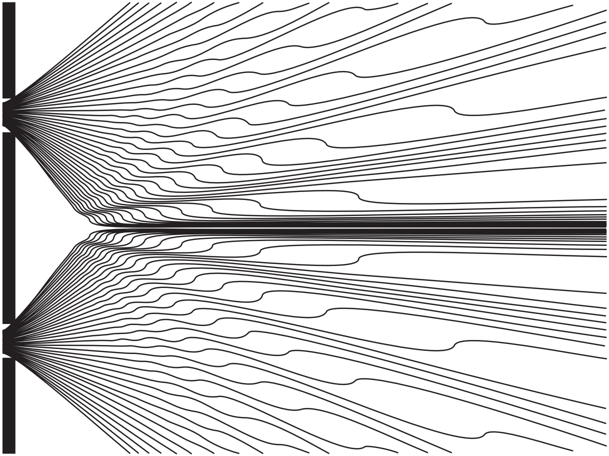
Werner Heisenberg in 1958:

"We can no longer speak of the behavior of the particle independently of the process of observation."
"The idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them [...], is impossible."

Well, Heisenberg was wrong. Bohmian mechanics is a counter-example to the impossibility claim.

W. Heisenberg (1901–1976)

But how can BM be compatible with Heisenberg's uncertainty relation?



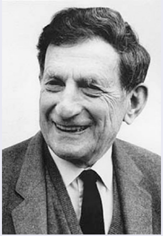
And how can BM be compatible with non-commuting observables?

Because different choices of the ready state ϕ of the apparatus lead to different kinds of interactions with the object.

Concrete example (Stern-Gerlach experiment) in Lecture 3.

History

- 1924: Einstein toys with the idea that photons may have trajectories obeying an equation of motion similar to that of Bohmian mechanics. John Slater joins him.
- 1926: Louis de Broglie discovers Bohmian mechanics, calls it "pilot-wave theory."
- 1945: Nathan Rosen (the R of EPR) independently discovers Bohmian mechanics.
- 1952: David Bohm independently discovers Bohmian mechanics. He is the first to realize that the theory is empirically adequate.



David Bohm (1917–1992)

Thank you for your attention