Mini Course on Interpretations of Quantum Mechanics

Lecture 2

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The quantum measurement problem

What the problem is

- The apparatus consists of electrons and quarks, so it should be possible to treat it like a quantum system with a wave fct ϕ on $\mathbb{R}^{3N}$, $N > 10^{23}$.
- If we do, then $\Psi(t_0) = \psi \otimes \phi$ evolves according to the Schrödinger eq. to $\Psi(t_1) = \sum_\alpha c_\alpha \Psi_\alpha$, where Ψ_α corresponds to a needle pointing to α. A superposition of different outcomes.
- $\Psi(t_1)$ doesn't say what the actual outcome is.
- We might have expected a state $\Psi(t_1)$ with a unique needle position.
- We might have expected a random state because the outcome should be random.

Schrödinger's cat

is a particular version of the problem. Schrödinger formulated it to criticize the Copenhagen interpretation of QM.

What exactly is the problem?

Bob: Superposition or not, $\Psi(t_1)$ still yields the right probabilities.

Alice: Everybody agrees about the empirically right probabilities. That is not the problem.

Bob: Then where is the problem?

Alice: The problem is about what is there in reality.

Bob: I believe there is no microscopic reality, that observables don't have values before the measurement.

Alice: But that is a hypothesis about reality, too. If only the wave function exists, that is a hypothesis about reality, too. The measurement problem puts constraints on the possible hypotheses about reality.

Bob: Which constraints?

Let's pin down the problem

3 assumptions

1. In each run of the experiment, there is a unique outcome.
2. The wave function is a complete description of a system's physical state in reality. (There are no further variables.)
3. The time evolution of the wave function of an isolated system, not entangled with the outside, is always given by the Schrödinger eq.

Together, they lead to a contradiction: By 3, $\Psi(t_1)$ is generically a superposition of Ψ_α corresponding to different outcomes. Thus, $\Psi(t_1)$ doesn't select an outcome. If there were further variables (such as Bohm's Q), they could select an outcome, but by 2 there aren't. Thus, there is no unique outcome, in contradiction to 1.

Consequence

We need to drop one of the 3 assumptions.

Bohmian mechanics drops 2, the GRW collapse theory 3, many-worlds 1.

Ways out of the problem?

Bob: As a consequence of decoherence, it is impossible for a second apparatus to distinguish between the superposition $\sum_\alpha c_\alpha \Psi_\alpha$ and a mixture of the Ψ_α with frequencies $|c_\alpha|^2$.

Alice: Yes, but the question is, whether in reality there is a superposition or a mixture. If it is a superposition, then it is not a mixture.

Bob: Nobody can actually solve the Schrödinger equation for 10^{23} interacting particles.

Alice: We don't need to. If Ψ_α looks like a state including a needle pointing to α then we know by linearity that $\Psi(t_0)$ evolves to $\Psi(t_1) = \sum c_\alpha \Psi_\alpha$, a superposition of macroscopically different states.

Bob: Systems (such as object and apparatus together) are never isolated.

Alice: The way to treat a non-isolated system is by regarding it as a subsystem of a bigger, isolated system, maybe the entire universe. So the problem persists.

Bob: Maybe there is no wave function of the universe.

Alice: You are welcome to propose hypotheses about what there is in reality. But you can't have all of 1,2,3.

Ways out of the problem?

Bob: Who knows whether the initial wave function is really a product as in $\Psi_{t_1} = \psi \otimes \phi$.

Alice: It is easy to verify that the problem persists if it is only approximately a product.

Bob: The collapse of the wave function resembles the collapse of a probability distribution: as soon as I have more information, such as $X \in B$, I have to update my probability distribution ρ_{t-} for X accordingly, namely to

$$\rho_{t+}(x) = 1_{x \in B}\, \rho_{t-}(x)\,.$$

Alice: If we insist that the wave function is complete, then there never is any new information, as there is nothing that we are ignorant of.

None of Bob's considerations seems to remove the problem.

Upshot

We need to drop one of the 3 assumptions.

The question that the discussion circles about

What is actually there in reality?

The GRW theory of spontaneous collapse

Spontaneous collapse: GRW theory

Key idea:

The Schrödinger equation is only an approximation, valid for systems with few particles ($N < 10^4$) but not for macroscopic systems ($N > 10^{23}$). The true evolution law for the wave function is non-linear and stochastic (i.e., inherently random) and avoids superpositions (such as Schrödinger's cat) of macroscopically different contributions.

Put differently, regard the collapse of ψ as a physical process governed by mathematical laws.

GianCarlo Ghirardi (1935–2018)

Explicit equations by Ghirardi, Rimini, and Weber [Phys.Rev. D 1986]

The predictions of the GRW theory deviate very very slightly from the quantum formalism. At present, no experimental test is possible.

GRW's stochastic evolution for ψ

- is designed for non-relativistic quantum mechanics of N particles
- meant to replace Schrödinger eq as a fundamental law of nature
- involves two new constants of nature:
  - $\lambda \approx 10^{-16}\,\mathrm{sec}^{-1}$, called collapse rate per particle.
  - $\sigma \approx 10^{-7}\,\mathrm{m}$, called collapse width.
- Def: ψ evolves as if an observer outside the universe made, at random times with rate $N\lambda$, quantum measurements of the position observable of a randomly selected particle with inaccuracy σ.
- "rate $N\lambda$" means that
  prob(an event in the next dt seconds) $= N\lambda\, dt$.
- more explicitly: Schrödinger evolution interrupted by jumps of the form

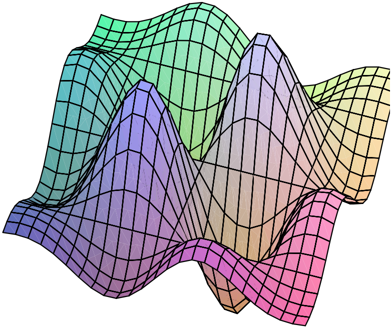
  $$\psi_{T+} = e^{-\frac{(\boldsymbol{q}_k - \boldsymbol{q})^2}{4\sigma^2}}\,\psi_{T-}\,,$$

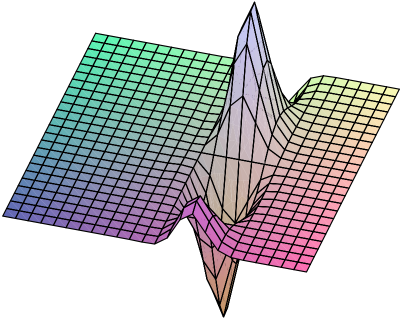
  i.e., multiplication by a Gauss function with random label k, center $\boldsymbol{q}$ and time T.
- $\mathrm{prob}(\boldsymbol{q} \in d^3\boldsymbol{q}) = \|\psi_{T+}\|^2 d^3\boldsymbol{q} = |\psi_{T-}(\boldsymbol{q}_k = \boldsymbol{q})|^2 * \text{Gaussian}$

GRW's spontaneous collapse

before the "spontaneous collapse":

and after:

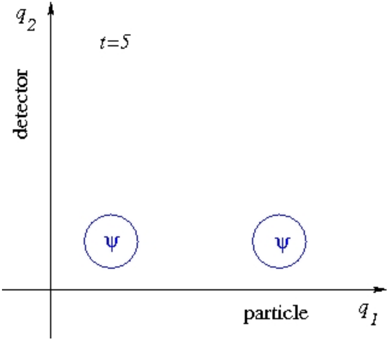




- In Hilbert space: piecewise deterministic stochastic jump process. ψ_t jumps at random times to random destinations.
- For a single particle, one collapse every 100 million years.
- For 10^4 particles, one collapse every 10,000 years.
- For 10^{23} particles, one collapse every 10^{-7} seconds.
- No-signaling theorem

How GRW theory solves the measurement problem

Evolution of ψ in configuration space of particle + detector:



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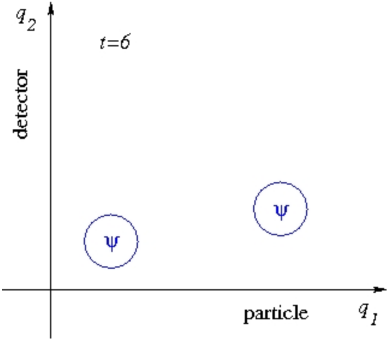

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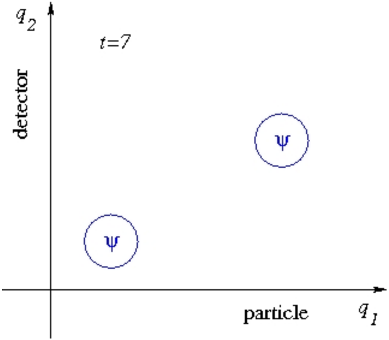

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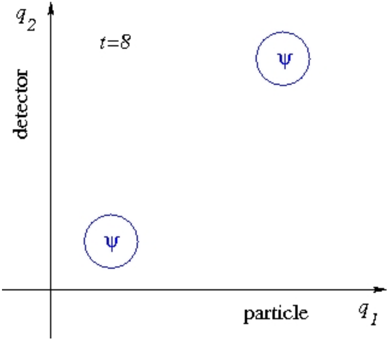

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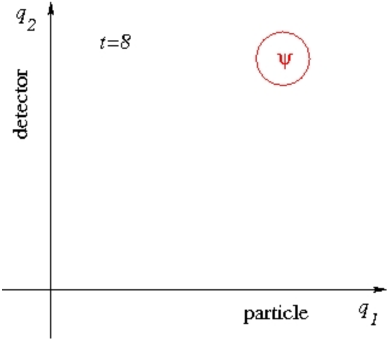
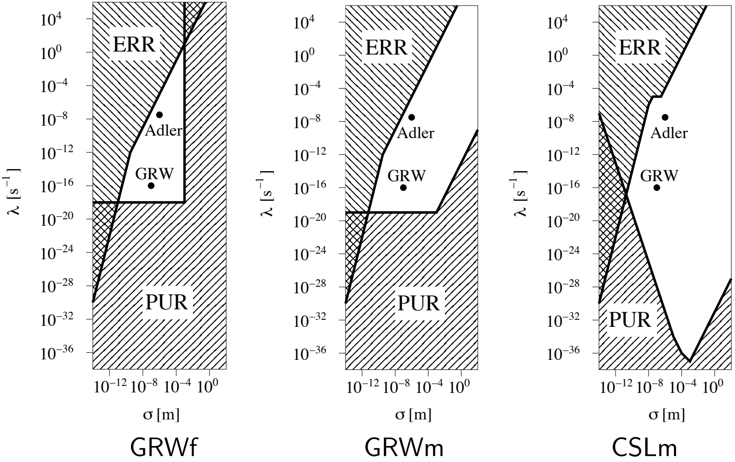


- As soon as a collapse occurs for one particle in the apparatus, the superposition in the test particle is gone as well.
- A macroscopic superposition $\sum_i \psi_i$ such as Schrödinger's cat would collapse within 10^{-7} seconds.
- It would collapse, up to tails of the Gaussian, to one of the macroscopically distinct wave packets ψ_i (to either $|\text{dead}\rangle$ or $|\text{alive}\rangle$).
- The probability that ψ collapses to ψ_i is, up to Gaussian tails, given by $\|\psi_i\|^2$.
- That is why GRW theory agrees with the standard quantum prediction to an excellent degree of approximation.

- But in principle, the predictions of GRW theory can differ from standard QM.
- For example, in a double slit experiment in which it takes the particle 300 million years to travel from the double slit to the screen, the interference pattern would disappear.
- It is not easy to test GRW against standard QM.
- Dramatic energy increase for much smaller σ values than 10^{-7} m
- Slight energy increase for $\sigma = 10^{-7}$ m

GRW theories are empirically adequate

Their predictions deviate very very slightly from the quantum formalism.



Parameter diagrams (log-log scale). ERR = empirically refuted region, PUR = philosophically unsatisfactory region [Feldmann, Tumulka arXiv:1109.6579]

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- Example: The ontology of Bohmian mechanics consists of space $\mathbb{E}^3$, time $\mathbb{E}^1$, particles Q, and a wave function ψ.

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It was long thought that the key to clarity in QM was to avoid talking about ontology and stick to operational statements. That thought has not paid off.

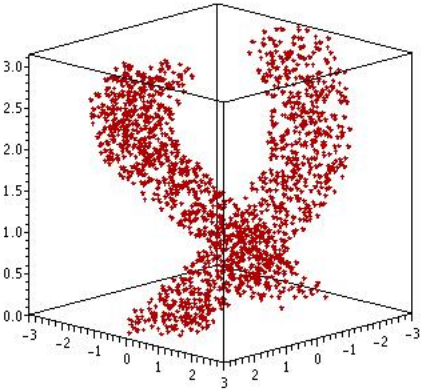
The word “beables”

- Bell coined the word as be-ables in contrast to observ-ables.
- Beables are the variables that represent the ontology (the things that exist), the quantities that actually have values (in contrast to observables).
- The word “beables” is also meant to suggest a tentative character (meaning “could be”) because what the beables are depends on the theory. Different theories have different pictures of what is real in the world.

The expression “primitive ontology”

- Def: Primitive ontology is the part of the ontology that represents matter in 3d space (or 4d space-time).
- Example: In Bohmian mechanics, the ontology consists of the particles and the wave function; the primitive ontology consists of the particles.
- I think that for GRW theory to make sense, it needs a primitive ontology.
- Here are two proposals: “flash ontology” and “matter density ontology.”

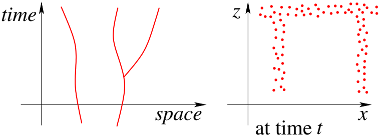
Flash ontology



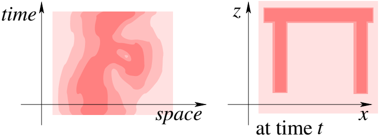
Instead of particle world lines, there are world points in space-time, called "flashes." A macroscopic object consists of a galaxy of flashes.

Ontologies

- Suppose a theory T talks a about particles in the literal sense, having world lines in space-time.



- Then we say that T has a particle ontology.
- Examples: Classical mechanics, Bohmian mechanics.
- Now suppose that a theory T' says that matter is continuously distributed in 4D space-time, with density function $m(t, \boldsymbol{x})$ [or $m_\mu(t, \boldsymbol{x})$ or $m_{\mu\nu}(t, \boldsymbol{x})$].



- Then we say that T' has a matter density ontology.

Laws for the primitive ontology

Def: GRWf [Bell 1987]

If ψ collapses at time T with center $\boldsymbol{q}$ then put a flash at $(T, \boldsymbol{q})$.

Def: GRWm [Diósi 1989; Ghirardi, Grassi, Benatti 1995; Goldstein 1998]

matter is continuously distributed with density given by

$$\begin{aligned} m(t, \boldsymbol{q}) &= \sum_{k=1}^{N} m_k \int \delta^3(\boldsymbol{q} - \boldsymbol{q}_k)\, |\psi_t(\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N)|^2 \, d^3\boldsymbol{q}_1 \cdots d^3\boldsymbol{q}_N \\ &= \langle \psi_t | \mathcal{M}(\boldsymbol{x}) | \psi_t \rangle \end{aligned}$$

with $\mathcal{M}(\boldsymbol{x}) = \sum_{k=1}^{N} m_k \, \delta^3(\boldsymbol{x} - \hat{\boldsymbol{Q}}_k)$ the mass density operators.

GRWf and GRWm are empirically equivalent (exercise).

Why we need a primitive ontology (1)

- There is a logical gap between saying

$$\text{“}\psi \text{ is the wave function of a live cat”} \tag{1}$$

  and saying

$$\text{“there is a live cat.”} \tag{2}$$

- After all, in Bohmian mechanics, (2) follows from (1) by virtue of a law of the theory, $Q_t \sim |\psi_t|^2$.
- Imagine Bohmian particles guided by a GRW wave function [Allori et al. arXiv:1206.0019]. The particles behave in a catastrophic way, although the wave function looks reasonable. So if you haven't specified the primitive ontology, you don't know what cats or pointers do.

Why we need a primitive ontology (2)

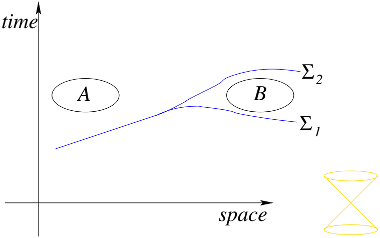
- Without it, paradoxes arise:
- **Paradox:** One might think GRW fails to solve the measurement problem: suppose
$$\psi = c_1 \, \psi_1 + c_2 \psi_2$$
is a superposition of macroscopically different states ψ_1, ψ_2. If $c_1 = \sqrt{0.5} = c_2$ then there is a problem; if $c_1 = \sqrt{0.4}$ and $c_2 = \sqrt{0.6}$ then there is still a problem. How small would c_1 have to be for the problem to disappear?
- **Answer:** The reasoning misses the primitive ontology. In GRWm, e.g., an *m* function close to that of a live cat is still an *m* function of a live cat.

Why we need a primitive ontology (3)

- Another one:
- **Paradox:** Suppose $|c_1|^2$ is near 1 and $|c_2|^2$ near 0. If we made a measurement of the macrostate, there is a positive probability $|c_2|^2$ for finding ψ_2. So how can we say that a cat with $\psi = c_1|\text{dead}\rangle + c_2|\text{alive}\rangle$ is really dead?
- **Answer:** In GRWm, "the cat is dead" means that the *m* function looks and behaves like a dead cat. The measurement might change ψ to near $|\text{alive}\rangle$, and then the cat *is* alive in GRWm. So, GRWm allows for resurrections—with tiny probability!

Why we need a primitive ontology (4)

A problem about relativistic facts for GRW∅:



Consider an EPR experiment, in which two particles in the singlet spin state are widely separated in space, and a Stern–Gerlach experiment is carried out on each particle. The reduced spin state ρ of particle A (obtained by tracing out the spin of particle B) will depend on the choice of hypersurface Σ: If $\Sigma = \Sigma_2$ lies after the experiment on particle B but before that on particle A, then ρ will be a pure state. If $\Sigma = \Sigma_1$ lies before both experiments, ρ will be mixed.

This poses a problem of finding a consistent relativistic specification of facts for GRW∅. However, the problem evaporates for GRWf/m.

Limitations to knowledge

Limitations to knowledge (1)

In any version of QM, you cannot measure the wave function.

- Example: Alice chooses a direction $\boldsymbol{a}$ in space and prepares a spin-$\frac{1}{2}$ particle with $\psi = |\text{spin up in } \boldsymbol{a}\rangle$.
- She hands it to Bob with the challenge to determine ψ (or $\boldsymbol{a}$).
- According to the rules of QM, Bob can do no better than perform a Stern-Gerlach experiment in a direction $\boldsymbol{b}$ of his choice and obtain 1 bit ("up" or "down").
- He can conclude whether $\boldsymbol{a}$ is more likely to lie in the hemisphere closer to $\boldsymbol{b}$ or closer to $-\boldsymbol{b}$, but cannot determine $\boldsymbol{a}$.
- (If the game is repeated and Alice always prepares the same ψ, Bob can determine ψ to desired accuracy. But not in a single run.)
- Nature knows in every single run what ψ is, or at least that an experiment in direction $\boldsymbol{a}$ must yield "up" with certainty. So,

Limitation to knowledge

Certain variables have well-defined values in the world (known to nature), although we cannot measure them, even with all future advances.

Limitations to knowledge (2)

Limitations to knowledge may seem to conflict with some principle of science, such as

> *"a statement is unscientific or even meaningless if it cannot be tested experimentally, an object is not real if it cannot be observed, and a variable is not well-defined if it cannot be measured."*

- But limitations to knowledge are a fact of quantum mechanics.
- Get used to them!
- The "principle" above is not a principle at all, it is wrong. It is exaggerated positivism.

Limitations to knowledge (3)

- are unknown in classical physics but common in quantum physics.
- Another example:
- Suppose $\{\varphi_1^a, \varphi_2^a\}$ and $\{\varphi_1^b, \varphi_2^b\}$ are two orthonormal bases of $\mathscr{H}$.
- Alice chooses either $i = a$ or $i = b$ and prepares an ensemble of particles, each in φ_1^i with prob $\frac{1}{2}$ and φ_2^i with prob $\frac{1}{2}$.
- The particles are handed over to Bob, who is asked whether $i = a$ or $i = b$.
- Claim: Bob can't find out empirically. The two ensembles are empirically indistinguishable.
- This is another limitation to knowledge: There is a fact whether in reality the ensemble is *a* or *b*. (Nature knows because Alice knows. She knows the state vector of every single particle, and she can prove it.)
- How to prove the claim?
- For every projection operator P, Bob's probability of the corresponding outcome is $\frac{1}{2}\langle\varphi_1^i|P|\varphi_1^i\rangle + \frac{1}{2}\langle\varphi_2^i|P|\varphi_2^i\rangle = \frac{1}{2}\mathrm{tr}P$. (Similar for POVMs.)

Limitations to knowledge in GRW theories

Can we measure, e.g., the number of collapses in a system (e.g., water droplet with 10^{15} molecules) during the time interval $[t_1, t_2]$? Is there a "Geiger counter for collapses"?

Short answer: no

In GRW theories, nature knows the exact number of collapses, but inhabitants can find it out only with macroscopic inaccuracy (say, $\pm 10^9$ per second).

[Cowan and Tumulka arXiv:1307.0810, 1307.0827, 1312.7321]

Thank you for your attention