Mini Course on Interpretations of Quantum Mechanics Lecture 3

Roderich Tumulka

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

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Many worlds

- Hugh Everett proposed a many-worlds theory in 1957.
- Less known, Erwin Schrödinger proposed a different many-worlds theory in 1925.

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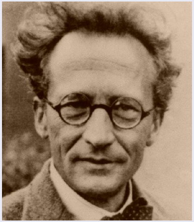
  where ψ evolves according to the Schrödinger equation.
- This is called Schrödinger's first theory (or "Sm").

Schrödinger's first theory (1925)

$$m(\boldsymbol{x}, t) = \sum_{k=1}^{N} \int_{(\mathbb{E}^3)^N} dq\, \delta^3(\boldsymbol{x} - \boldsymbol{q}_k)\, |\psi(q,t)|^2$$



E. Schrödinger (1887–1961)

He soon abandoned this theory because he thought it made wrong predictions. But actually, it makes correct predictions, but it has a many-worlds character.

For Schrödinger's cat, $\psi = \frac{1}{\sqrt{2}}\psi_{\text{dead}} + \frac{1}{\sqrt{2}}\psi_{\text{alive}}$,
it follows that $m = \frac{1}{2} m_{\mathrm{dead}} + \frac{1}{2} m_{\mathrm{alive}}$.

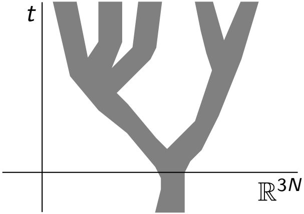
There is a dead cat and a live cat, but they are like ghosts to each other (they do not notice each other), as they do not interact. So to speak, they live in parallel worlds. [Allori et al. 0903.2211]

Not knowing about Schrödinger's proposal, Everett advocated a many-worlds view in 1957, but with a different ontology: only ψ exists, no beables in 3-space ("S∅").

Motivation behind S∅

In a measurement process, the wave function branches as in $\Psi(t_1) = \sum_\alpha c_\alpha \Psi_\alpha$. Maybe this means that for each α there exists an apparatus with needle pointing to α.
Configurations in the support of Ψ_α look like normal situations that don't "notice" any other branches. So maybe other branches exist and we don't notice.

Hugh Everett (1930–1982)



How Sm and S∅ solve the measurement problem

Assumption 1 of the measurement problem said there is a unique outcome. This is dropped if every possible outcome is realized in some part of reality.

Limitation to knowledge

People often say many-worlds is bad b/c you can't test the existence of the other worlds. I think that is a bad reason, based on insufficient appreciation that all versions of QM entail limitations to knowledge. Note that it *follows* from the Schrödinger eq. that no branch "notices" the other branches.

Sm vs S$\emptyset$

Issue of ontology

- Sm is way easier to understand and think through than S$\emptyset$ because in Sm cats and trees and rocks are part of the matter in 3-space, whereas in S$\emptyset$ there is nothing in 3-space. In S$\emptyset$, we would have to accept that the statement "the cat is alive" really means that the wave function in concentrated in the Hilbert subspace corresponding to live cats.
- As in GRW$\emptyset$, there seems to remain a logical gap in S$\emptyset$: The mere fact that a function on configuration space is nonzero at q doesn't mean that there is a world with configuration q. (Think of the potential $V(q)$ of classical mechanics.)
- Bohmian mechanics illustrates how ψ can have many branches without corresponding worlds.

Preferred-basis problem

People often say many-worlds is bad b/c it requires a preferred basis but nothing in the theory (the Hilbert space $\mathscr{H}$, the Hamiltonian H, the state vector Ψ) selects such a basis.

I think that is a bad reason. In Sm, the law for m selects the position basis. Even in S$\emptyset$, one could add a law saying that Ψ is a field on configuration space $\mathbb{R}^{3N}$, and this law would select the position basis.

Probability in MW

People often say that many-worlds is bad b/c probabilities don't make sense if every outcome is realized.

That's a more subtle issue.

Probability in MW

MW theories (Sm and S∅) imply that the statistics predicted by the rules of quantum mechanics will be observed in most worlds (i.e., they make correct predictions), provided that we count worlds with $|\Psi|^2$ weights.

Example

1000 spin-z measurements on particles with $\psi = \sqrt{2/3}|\text{up}\rangle + \sqrt{1/3}|\text{down}\rangle$ result in 2^{1000} branches Ψ_α with $\alpha \in \{\text{up}, \text{down}\}^{1000}$. Frequency of "up" in α is $f(\alpha) = \frac{1}{1000}\sum_{i=1}^{1000} 1_{\alpha_i=\text{up}}$. Most sequences α have $0.49 < f(\alpha) < 0.51$. But

$$\sum_{\alpha:0.66<f(\alpha)<0.67} \|\Psi_\alpha\|^2 > 0.99\,.$$

The question is

Is it possible for a physical theory to introduce a law prescribing how to count worlds?

I am skeptical.

Other proposals for justifying probability in MW

- David Deutsch [quant-ph/9906015]: It is rational for inhabitants of a multiverse to behave as if the events they perceive were random with probabilities given by the Born rule. (However, this doesn't explain why we see frequencies in agreement with Born's rule.)
- Lev Vaidman [quant-ph/9609006]: In a multiverse, I can be ignorant of which world I am in, and express my ignorance as a probability distribution. (However, it is not clear why one distribution would be "correct" and others not.)

The Copenhagen Interpretation

Introducing the CI

- It is hard to explain or define the CI. While BM, GRWf, GRWm, GRW∅, Sm, S∅ have few basic laws, and all other claims follow from the basic laws, CI has many "views."
- What is real according to the CI? Presumably, wave functions for micro-objects and the classical macro-state for macro-objects. (However, that's problematical because the concept of "macroscopic" is vague by its nature.)
- How does the CI solve the measurement problem? Frankly, I don't see that it does.

Positivism

"a statement is unscientific or even meaningless if it cannot be tested experimentally, an object is not real if it cannot be observed, and a variable is not well-defined if it cannot be measured."

CI leans towards positivism. In the words of Heisenberg (1958):

"The idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them [...], is impossible."

Feynman (1962) did not like that:
"Does this mean that my observations become real only when I observe an observer observing something as it happens? This is a horrible viewpoint. Do you seriously entertain the thought that without observer there is no reality? Which observer? Any observer? Is a fly an observer? Is a star an observer? Was there no reality before 10^9 B.C. before life began? Or are you the observer? Then there is no reality to the world after you are dead? I know a number of otherwise respectable physicists who have bought life insurance."

Richard Feynman (1918–1988)

Complementarity

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Bell commented (1986):

"What hope then for the rest of us?"

How Bohr defined complementarity:

"Any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena."

How I understand Bohr's idea:

In order to compute a quantity of interest (e.g., the wave length of light scattered off an electron), we use both Theory A (e.g., classical theory of billiard balls) and Theory B (e.g., classical theory of waves) although A and B contradict each other. It is impossible to find one Theory C that replaces both A and B and explains the entire physical process. Instead, we should leave the conflict between A and B unresolved and accept the idea that reality is paradoxical.

- According to key elements of the Copenhagen view,
  - reality itself is contradictory.
  - That is why there is no Theory C, no single picture that completely describes reality.
  - At the same time, we can never observe a contradiction in experiment (e.g., because we can only observe one of two complementary observables).
  - And since we cannot observe contradictions, the contradictions are somehow not a problem. (My understanding of Bohr)

According to Copenhagen, it is like in the Charles Addams cartoon:

We never see the paradoxical thing happen. But we see traces showing that it must have happened.

Spin

The Pauli equation

Consider N spin-$\frac{1}{2}$ particles.

$\psi(t) : \mathbb{R}^{3N} \to (\mathbb{C}^2)^{\otimes N}$,
$\psi = \psi_{s_1 \ldots s_N}(\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N, t)$

The appropriate version of the Schrödinger eq is the Pauli eq

$$i\hbar \frac{\partial \psi}{\partial t} = \sum_{k=1}^{N} \frac{1}{2m_k} \Big(-i\hbar \nabla_{\boldsymbol{q}_k} - \boldsymbol{A}(\boldsymbol{q}_k) \Big)^2 \psi - \sum_{k=1}^{N} \frac{\hbar}{2m_k} \boldsymbol{\sigma}_k \cdot \boldsymbol{B}(\boldsymbol{q}_k) \psi + V\psi$$

with $\boldsymbol{A}$ the vector potential, $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ the magnetic field, V the electric potential, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ the Pauli matrices, and $\boldsymbol{\sigma}_k$ acting on s_k.

You might have expected that Bohm needs little spinning balls. But it is much easier.

Equation of motion: [Bell 1966]

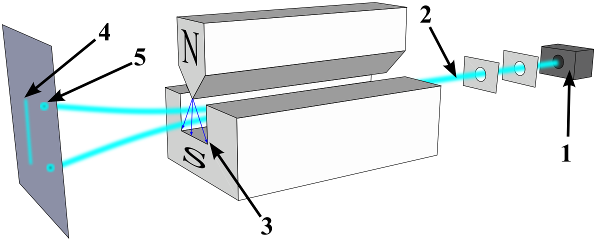
$$\frac{d\boldsymbol{Q}_k(t)}{dt} = \frac{\boldsymbol{j}_k}{\rho}(Q(t),t) = \frac{\hbar}{m_k}\mathrm{Im}\frac{\psi^*\nabla_k\psi}{\psi^*\psi}(Q(t),t)$$

where $\phi^*\psi = \sum_{s=1}^{2^N} \phi_s^*\psi_s$ inner product in spin-space, $s=(s_1...s_N)$.

So the electron is still a point, and not spinning. Spin is merely in the wave fct (the wave fct is spinor-valued).

The Stern-Gerlach experiment

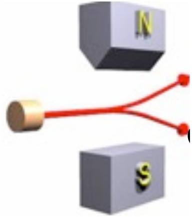
Then what does a “spin measurement” do in BM?



Picture credit: `http://en.wikipedia.org/wiki/Stern-Gerlach_experiment`

Otto Stern, Walther Gerlach 1922

The Stern-Gerlach experiment in Bohmian mechanics



Stern–Gerlach experiment

Wave packet $\psi = \binom{\psi_\uparrow}{\psi_\downarrow}$ splits into two packets, one purely $\uparrow$, the other purely $\downarrow$. Then detect the position of the particle: If it is in the spatial support of the $\uparrow$ packet, say that the outcome is "up."

So, the "measurement" is not literally a measurement (i.e., not a determination of a pre-existing value). The outcome is a random value generated in the experiment. That is common with "quantum measurements" in Bohmian mechanics (or GRW or MW), except for position measurements.

Prediction

Since $Q \sim |\psi|^2$, Prob(up) $= \|\psi_\uparrow\|^2$ and Prob(down) $= \|\psi_\downarrow\|^2$
Empirically correct. Same in direction $\boldsymbol{a} \in \mathbb{R}^3$. In particular, BM is compatible with non-commuting operators $\boldsymbol{a} \cdot \boldsymbol{\sigma}$.

Actual spin vector?

Some authors felt electrons should have an actual spin vector. Here is why the most natural proposal [Bohm-Schiller-Tiomno 1955] is unconvincing.

- Consider 1 electron.
- state (ontology) is $(\psi(t), \boldsymbol{Q}(t), \boldsymbol{S}(t))$
  with $\psi(t) : \mathbb{R}^3 \to \mathbb{C}^2$, $\boldsymbol{Q}(t) \in \mathbb{R}^3$, and actual spin vector

$$\boldsymbol{S} = \frac{\psi^* \boldsymbol{\sigma} \psi}{\psi^* \psi}(t, \boldsymbol{Q}(t)) \in \mathbb{R}^3 \,.$$

- One might expect that the outcome of the Stern-Gerlach experiment is the z-component of $\boldsymbol{S}$. It's not!
- The outcome is a function of $\boldsymbol{Q}$ and ψ, and their equations of motion don't depend on $\boldsymbol{S}$, so $\boldsymbol{S}$ has no influence on the outcome.
- $\boldsymbol{S}$ is superfluous.

No-hidden-variable theorems

"Hidden variable" can mean

- Any further variable assumed to exist in addition to ψ (such as Q in BM, which however is not hidden at all!)
- The assumption that every observable has an actual value already before a quantum measurement. (Not the case in BM.)

Let me discuss on the latter view, which is suggested by the terminology of "observable" and "measurement." Let us suppose that with every self-adjoint operator A there is associated a physical quantity v_A, the actual value of the observable A, and that a quantum measurement of A simply reveals the value v_A. Can it be this way?

There are several no-hidden-variable (NHV) theorems. We focus on the most important one, first proved by Gleason (1957), then in other/simpler ways by Specker (1960), Bell (1966), Kochen and Specker (1967), Mermin (1990), and Peres (1991).

NHV theorem

Suppose $3 \leq \dim \mathscr{H} < \infty$. Let $\mathscr{A}$ be the set of all self-adjoint operators on $\mathscr{H}$, and fix a $\psi \in \mathscr{H}$ with $\|\psi\| = 1$. The Born distribution for $A \in \mathscr{A}$ is

$$\text{Prob}(\alpha) = \|P_\alpha \psi\|^2 = \langle \psi | P_\alpha \psi \rangle \tag{1}$$

for $A = \sum_\alpha \alpha P_\alpha$. For pairwise-commuting A, B, C with $A = \sum_{\alpha\beta\gamma} \alpha P_{\alpha\beta\gamma}$, $B = \sum_{\alpha\beta\gamma} \beta P_{\alpha\beta\gamma}$, $C = \sum_{\alpha\beta\gamma} \gamma P_{\alpha\beta\gamma}$, the joint Born distribution is

$$\text{Prob}(\alpha, \beta, \gamma) = \|P_{\alpha\beta\gamma} \psi\|^2 . \tag{2}$$

NHV theorem

Consider a joint distribution of random variables v_A for all $A \in \mathscr{A}$. Suppose that a quantum measurement of any $A \in \mathscr{A}$ yields v_A. Suppose further that whenever $A, B \in \mathscr{A}$ commute, then a quantum measurement of A doesn't change the value of v_B (nor that of v_A). Then the joint distribution of v_A, v_B, v_{A+B} disagrees with the joint Born rule (2).

Proof: Make measurements of $A, B, A+B$ with $AB = BA$. Then the eigenvalues of $A+B$ are of the form $\alpha+\beta$ with α and β eigenvalues of A and B. The measurements don't change v_A, v_B, v_{A+B}. The joint Born rule would imply that $v_{A+B} = v_A + v_B$. But

Lemma (proof omitted)

For $3 \leq \dim \mathscr{H} < \infty$, there is no mapping $v : \mathscr{A} \to \mathbb{R}$ with the two properties that

- $v_A \in \text{spectrum}(A)$ for all $A \in \mathscr{A}$
- whenever $AB = BA$, then $v_{A+B} = v_A + v_B$.

□

Upshot

It's not convincingly possible that there is an actual value v_A for every observable A.

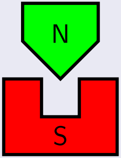
But BM is deterministic...

...so the outcome Z of an experiment is a function $f(X, Y, \psi, \phi)$ of the initial data at t_0, $\Psi(t_0) = \psi \otimes \phi$ and $Q(t_0) = (X, Y)$.

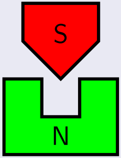
Why isn't Z a v_A?

Because it depends on Y and ϕ, not just on A.

Example: Two experiments that are quantum measurements of σ_z



One is a Stern-Gerlach experiment in the z direction.



The other uses a magnet with inverted polarity and calls the outcome "down" if the particle is found in the upper packet.

On the same X and ψ, the two experiments sometimes give different results. ("contextuality")

> “The result of an experiment depends on the experiment.”

[Dürr, Goldstein, Zanghì 2004]

...and not just on A. Different experiments belonging to the same observable may yield different results but the same probability distribution of results.

Quantum logic

“Quantum logic” can mean

- a certain piece of mathematics that is rather pretty;
- a certain analogy between two formalisms that is rather limited;
- a certain philosophical idea that is rather misguided.

Let me explain.

Logic is the collection of those statements and rules that are valid in every conceivable universe and every conceivable situation.

Consider rules of logic for "and," "or," "not":

Definition: Boolean algebra

Set $\mathscr{A}$ of elements $A, B, C, \ldots$ of which we can form $A \wedge B$, $A \vee B$, and $\neg A$, such that the following rules hold:

- $\wedge$ and $\vee$ are associative, commutative, and idempotent ($A \wedge A = A$ and $A \vee A = A$).
- Absorption laws: $A \wedge (A \vee B) = A$ and $A \vee (A \wedge B) = A$.
- There are elements $0 \in \mathscr{A}$ ("false") and $1 \in \mathscr{A}$ ("true") such that for all $A \in \mathscr{A}$, $A \wedge 0 = 0$, $A \wedge 1 = A$, $A \vee 0 = A$, $A \vee 1 = 1$.
- Complementation laws: $A \wedge \neg A = 0$, $A \vee \neg A = 1$.
- Distributive laws: $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$ and $A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$.

Agrees with rules of logic for $\wedge =$ and, $\vee =$ or, $\neg =$ not, and $A, B, C =$ statements.
Also applies to subsets A, B, C of some set Ω with $\wedge = \cap$, $\vee = \cup$, $\neg A = \Omega \setminus A$.

Now let

- $A, B, C, \ldots$ be (closed) subspaces of a Hilbert space $\mathscr{H}$
- $A \wedge B = A \cap B$
- $A \vee B = \overline{\text{span}}(A \cup B)$
- $\neg A = A^\perp = \{\psi \in \mathscr{H} : \langle\psi|\phi\rangle = 0 \,\forall \phi \in A\}$ (orthogonal complement of A)
- $0 = \{0\}$
- $1 = \mathscr{H}$ the full subspace.

Then all axioms except distributivity are satisfied. This is called an *orthomodular lattice* or simply *lattice*. Hence, a distributive lattice is a Boolean algebra, and the closed subspaces form a non-distributive lattice $\mathbb{L}(\mathscr{H})$.

Pretty mathematics.

The analogy I mentioned

- $\mathbb{L}(\mathscr{H}) \quad \leftrightarrow \quad$ Boolean algebra of logic
- Both are orthomodular lattices.
- Some authors call $A \in \mathbb{L}(\mathscr{H})$ a "proposition" or "yes-no question" (because you could do a quantum measurement of the projection operator to A).
- some authors call $\mathbb{L}(\mathscr{H})$ "quantum logic."

Why the analogy is rather limited

- For a spinor $\psi \in \mathbb{C}^2$, consider "ψ *lies in* $\mathbb{C}|\text{up}\rangle$" $=: \mathscr{P}$. Sounds like a proposition. Its negation is "ψ *lies in* $\mathscr{H} \setminus \mathbb{C}|\text{up}\rangle$", whereas $\mathbb{C}|\text{up}\rangle^\perp = \mathbb{C}|\text{down}\rangle$. (The negation of "spin is up" is not "spin is down," but "spin is in any direction but up.")
- In Wheeler's delayed-choice experiment without plate in the interference region. The subspace A of waves passing through the upper slit evolves to the subspace A' of waves arriving at the lower counter. It is natural to identify A with A' and with the proposition "*The particle passed through the upper slit.*" But that means to commit Wheeler's fallacy.

The philosophical idea I mentioned

is that logic as we know it is false, and that a different kind of logic with different rules applies in quantum physics—a *quantum logic*.

Why I call that misguided

Because logic is, by definition, what is true in every conceivable situation. It cannot depend on physical laws and cannot be revised by empirical science.

"There is no point in arguing with somebody who does not believe in logic."

Tim Maudlin (1958–)

Thank you for your attention