Mini Course on Interpretations of Quantum Mechanics
Lecture 4

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The Einstein-Podolsky-Rosen (1935) argument

The EPR argument (in a version due to Bohm 1951)

Claim: There are additional variables beyond the wave function.

Consider 2 spin-$\frac{1}{2}$ particles at spacelike separation with "singlet" spin state

$$\psi = \frac{1}{\sqrt{2}}\Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\Big) .$$

Alice measures σ_z of particle 1, obtains $A = +1$ (with prob $1/2$) or $A = -1$ (with prob $1/2$). ψ collapses to

$$\psi' = |\uparrow\downarrow\rangle \text{ if } A = +1 \text{ or } \psi' = |\downarrow\uparrow\rangle \text{ if } A = -1.$$

Bob measures σ_z of particle 2, obtains $B = -A$ with prob 1. It follows that particle 2 had a definite σ_z value already before Bob's measurement.

Locality assumption

Spacelike separated events cannot influence each other. In particular, Alice's measurement cannot change the physical situation on Bob's side.

It follows that Bob's particle had a definite σ_z value even before Alice's measurement, although ψ is not an eigenstate of $I \otimes \sigma_z$. Quod erat demonstrandum.

The original 1935 EPR argument

Claim: There are additional variables beyond the wave function.

Consider 2 spin-$\frac{1}{2}$ particles in 1d with entangled wave fct

$$\psi(x_1, x_2) = \delta(x_1 - x_2 + x_0)$$

with x_0 a (large) constant. Alice measures the position of particle 1, obtains X_1 (uniformly distributed). ψ collapses to

$$\psi'(x_1, x_2) = \delta(x_1 - X_1)\,\delta(X_1 - x_2 + x_0)\,.$$

Bob, spacelike separated from Alice, measures the position of particle 2 and obtains $X_2 = X_1 + x_0$ with prob 1. It follows that particle 2 had a definite position already before Bob's measurement.

Locality assumption

Spacelike separated events cannot influence each other. In particular, Alice's measurement cannot change the physical situation on Bob's side.

It follows that Bob's particle had a definite position even before Alice's measurement, although ψ is not a position eigenstate. Quod erat demonstrandum.

Confusion about the EPR argument

- In their paper, EPR went on to consider momentum measurements and argue that by measuring position on particle 1 and momentum on particle 2, you could know both the position and the momentum of each particle.
- But that distracted most readers from the central claim and its argument. Bohr replied only to the second part.
- In QM, we often practice not to think about physical reality but to focus on observable predictions. But you can't understand the argument unless you think about physical reality.

Too good to be true

The EPR argument is actually correct.

Upshot

Locality $\Rightarrow$ hidden variables

Since

$$
\begin{aligned}
&\tfrac{1}{\sqrt{2}}\Big(\big|{\uparrow\downarrow}\big\rangle - \big|{\downarrow\uparrow}\big\rangle\Big) \\
&= \tfrac{1}{\sqrt{2}}\Big(\big|{\rightarrow\leftarrow}\big\rangle - \big|{\leftarrow\rightarrow}\big\rangle\Big) \\
&= \tfrac{1}{\sqrt{2}}\Big(\big|\boldsymbol{a}\text{-up}\big\rangle\big|\boldsymbol{a}\text{-down}\big\rangle - \big|\boldsymbol{a}\text{-down}\big\rangle\big|\boldsymbol{a}\text{-up}\big\rangle\Big)
\end{aligned}
$$

up to a phase for every direction $\boldsymbol{a} \in \mathbb{R}^3$, the same argument provides the existence of a hidden variable for every $\boldsymbol{a} \cdot \boldsymbol{\sigma}$.

However, from the no-hidden-variable theorems, we know that that can't be.

Bell's nonlocality theorem

Nonlocality in Bohmian mechanics

$\frac{d\boldsymbol{Q}_1}{dt}$ depends on $\boldsymbol{Q}_2(t)$, no matter the distance $|\boldsymbol{Q}_1(t) - \boldsymbol{Q}_2(t)|$.

(And correspondingly, BM doesn't contain hidden variables for $\boldsymbol{a} \cdot \boldsymbol{\sigma}$.)

Nonlocality

Bell's nonlocality theorem (1964)

Some predictions of QM are incompatible with locality, regardless of which interpretation of QM is used. Put differently, certain statistics of outcomes (predicted by QM) are possible only if spacelike separated events sometimes influence each other.

These statistics were confirmed in experiment [Aspect 1982 etc.].

John Bell (1928–1990)

Bell's lemma (1964)

Non-contextual hidden variables cannot reproduce the statistics predicted by QM for certain experiments.

Upshot of EPR's argument

Locality implies the existence of non-contextual hidden variables for all local observables.

Note: EPR + Bell's lemma $\Rightarrow$ Bell's theorem

Bell's proof (1)

Again singlet state

$$\psi = \frac{1}{\sqrt{2}}\Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\Big)$$

Alice chooses arbitrary direction $\boldsymbol{a} \in \mathbb{R}^3$, $|\boldsymbol{a}| = 1$, and measures $\boldsymbol{a} \cdot \boldsymbol{\sigma}$ on her particle. Bob chooses $\boldsymbol{b}$ and measures $\boldsymbol{b} \cdot \boldsymbol{\sigma}$. QM predicts the following probabilities ($\boldsymbol{a} \cdot \boldsymbol{b} = \cos\theta$):

	$+$	$-$
$+$	$\frac{1}{2}\sin^2(\theta/2)$	$\frac{1}{2}\cos^2(\theta/2)$
$-$	$\frac{1}{2}\cos^2(\theta/2)$	$\frac{1}{2}\sin^2(\theta/2)$

Bell's proof (2)

By locality and EPR, Alice and Bob's outcome must be $v_{1\boldsymbol{a}} = v_{\boldsymbol{a}\cdot\boldsymbol{\sigma}\otimes I}$ and $v_{2\boldsymbol{b}} = v_{I\otimes\boldsymbol{b}\cdot\boldsymbol{\sigma}}$. These variables have the properties

- $v_{i\boldsymbol{a}} = \pm 1$
- $v_{2\boldsymbol{a}} = -v_{1\boldsymbol{a}}$

Now consider 3 directions $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$ for $\boldsymbol{a}$ or $\boldsymbol{b}$. Then

$$\begin{aligned} & \mathbb{P}\Big(v_{1\boldsymbol{\alpha}} = v_{1\boldsymbol{\beta}} \text{ or } v_{1\boldsymbol{\beta}} = v_{1\boldsymbol{\gamma}} \text{ or } v_{1\boldsymbol{\gamma}} = v_{1\boldsymbol{\alpha}}\Big) = 1 \\ \Rightarrow\quad & \mathbb{P}\big(v_{1\boldsymbol{\alpha}} = v_{1\boldsymbol{\beta}}\big) + \mathbb{P}\big(v_{1\boldsymbol{\beta}} = v_{1\boldsymbol{\gamma}}\big) + \mathbb{P}\big(v_{1\boldsymbol{\gamma}} = v_{1\boldsymbol{\alpha}}\big) \geq 1 \\ \Rightarrow\quad & \mathbb{P}\big(v_{1\boldsymbol{\alpha}} \neq v_{2\boldsymbol{\beta}}\big) + \mathbb{P}\big(v_{1\boldsymbol{\beta}} \neq v_{2\boldsymbol{\gamma}}\big) + \mathbb{P}\big(v_{1\boldsymbol{\gamma}} \neq v_{2\boldsymbol{\alpha}}\big) \geq 1 \end{aligned}$$

but the QM prediction is $\mathbb{P}(A \neq B) = \cos^2(\theta/2)$, which yields $3/4$ for the left-hand side if the angles between $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$ are all $120°$. $\square$

Misunderstandings about Bell's theorem

It is sometimes reported that Bell proved local hidden variables impossible. That is true but misses the point. That is Bell's lemma, not Bell's theorem. Bell's theorem says that any local theory is impossible.

Part 1 (EPR):	QM + locality	$\Rightarrow$	P
Part 2:	QM	$\Rightarrow$	not P
Conclusion:	QM	$\Rightarrow$	not locality

(P = existence of non-contextual hidden variables)

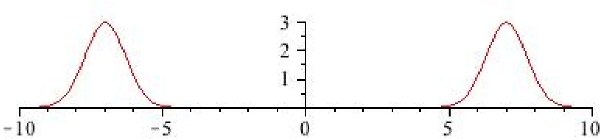
It is also sometimes reported that Bell disproved "local realism," and that we can choose to abandon either locality or realism. That's not true. There is no alternative to abandoning locality.

- Bohmian mechanics is nonlocal.
- GRW is nonlocal.
- Many-worlds is nonlocal.

It may seem that nonlocality conflicts with relativity, but we will see that this is not so.

Einstein's boxes argument

[Einstein ∼ 1927, unpublished], [Norsen: "Einstein's boxes" *Am. J. Phys.* 2005]



The wave function of a particle is half in a box in Paris and half in a box in Tokyo. Apply detectors to both boxes at time t (in some Lorentz frame)—at spacelike separation. One and only one detector clicks. If it is assumed that there was no fact about "where the particle actually is" before the detectors were applied, then this effect is nonlocal.

- Einstein intended this as an argument against the Copenhagen camp.
- The argument shows that any collapse theory is nonlocal.

Sm is nonlocal (1)

[Allori et al. 2011]

You might think Sm is local because of the following fact:

$m(t, \boldsymbol{x})$ in B does not depend on external fields in A or on the quantum state in A (it is a function of the reduced density matrix $\rho_B = tr_A|\psi\rangle\langle\psi|$ with ψ including apparatus).

I conclude that nothing that Alice can do in A, nor any events in A, can influence $m(t, \boldsymbol{x})$ in B. And yet, Sm is nonlocal:

Consider Einstein's boxes at a time t after applying detectors on both sides. The possible outcomes are 01 and 10. The wave function $\psi = \psi_t$ of the universe is

$$\psi = \psi_{01} + \psi_{10} \,,$$

and correspondingly,

$$m = m_{01} + m_{10} \,.$$

Thus the world in which Alice's result is 1 is the same world as the one in which Bob's result is 0—a fact created in a nonlocal way.

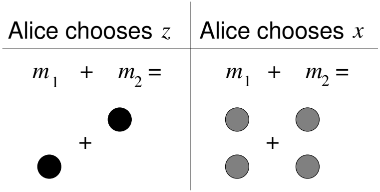
Sm is nonlocal (2)

- The m function alone, while revealing that there are two worlds in A and two worlds in B, does not encode the information conveying which world in A is the same as which world in B. That is, the pairing of worlds cannot be read off from $m(t, \cdot)$ even though it is an objective fact of Sm at time t, defined by means of the wave function ψ_t.
- Thus, the fact that Alice cannot influence m in B does not mean locality.
- One should suspect that Sm is nonlocal already when noticing that Sm involves a nonlocal object ψ and cannot (in any obvious way) be formulated without mentioning such an object.

Sm is nonlocal (3)

Moreover, even though Alice cannot influence the PO in B, she can influence other physical facts pertaining to B.

Consider a Bell experiment (with 2 electrons starting in the singlet state) in which Alice chooses either the x or the z direction for her magnet, while Bob always chooses the z direction. Suppose that at time t (in a certain Lorentz frame), Alice's detector has clicked but Bob's has not, although Bob's particle has already passed Bob's magnet. One finds that, in the region of Bob's particle,

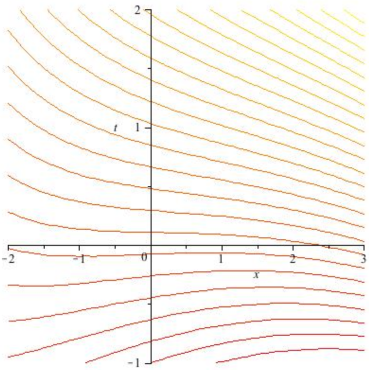


While $m(x)$ for $x \in B$ is unaffected by Alice's choice, each $m_\ell(x)$ is affected.

Bohmian mechanics in relativistic space-time

Bohmian mechanics in relativistic space-time

- If a preferred foliation (= slicing) of space-time into spacelike hypersurfaces ("time foliation" $\mathcal{F}$) is permitted, then there is a simple, convincing analog of Bohmian mechanics, $\mathrm{BM}_{\mathcal{F}}$. [Dürr et al. 1999] Without a time foliation, no version of Bohmian mechanics is known that would make predictions anywhere near quantum mechanics. (And I have no hope that such a version can be found in the future.)



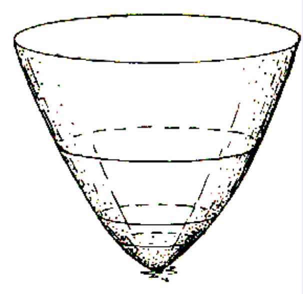
What does it mean for a theory to be relativistic?

Maybe there is no single property of a theory that can be regarded as "being relativistic." Rather, there are several relevant properties:

1. **Lorentz invariance.** Any Lorentz transform of any solution is another solution.
   - Can be made true trivially (e.g., for non-rel. theory) without changing predictions. Thus, necessary but not sufficient for anybody's notion of being relativistic.
2. **Commutation.** Field operators $[\phi(x), \phi(y)] = 0$ for spacelike separated x, y.
   - Easy to satisfy, seems not sufficient for being relativistic.
3. **No signaling faster than light.** Necessary but not sufficient.
4. **Locality à la EPR and Bell.** Violated in nature.
5. **No additional structure.** Don't introduce $\mathcal{F}$, use only $g_{\mu\nu}$ and ψ.
   - It seems possible to define foliations from $g_{\mu\nu}$ and/or ψ.
6. **Microscopic parameter independence.** If regions A, B are spacelike separated, then $\mathbb{P}(PO_A|\Phi_A, \Phi_B, \lambda) = \mathbb{P}(PO_A|\Phi_A, \lambda)$ for external fields Φ and hidden variables λ.
   - True in relativistic GRWf and GRWm, false in $\text{BM}_{\mathcal{F}}$.

Perhaps, the semantic question what we should mean by "relativistic" is irrelevant. The possibility seems worth considering that our universe has a time foliation.

Simplest choice of time foliation $\mathcal{F}$



Let $\mathcal{F}$ be the level sets of the function
T : space-time $\to \mathbb{R}$,
$T(x) = \text{timelike-distance}(x, \text{big bang})$.

E.g., $T(\text{here-now}) = 13.7$ billion years

Drawing: R. Penrose

Alternatively, $\mathcal{F}$ might be defined in terms of the quantum state vector ψ, $\mathcal{F} = \mathcal{F}(\psi)$ [Dürr, Goldstein, Norsen, Struyve, Zanghì 2014]

Or, $\mathcal{F}$ might be determined by an evolution law (possibly involving ψ) from an initial time leaf.

Bohmian mechanics for a single Dirac particle

No time foliation needed in this case.

Dirac equation:

$$i\hbar\gamma^\mu\partial_\mu\psi = m\psi \qquad \text{or} \qquad i\hbar\frac{\partial\psi}{\partial t} = -i\hbar\boldsymbol{\alpha}\cdot\nabla\psi + m\beta\psi$$

Equation of motion:

$$\frac{dX^\mu}{ds} \propto \overline{\psi}(X^\nu(s))\,\gamma^\mu\,\psi(X^\nu(s))$$

or, equivalently,

$$\frac{d\boldsymbol{X}}{dt} = \frac{\psi^*\boldsymbol{\alpha}\,\psi}{\psi^*\psi}(\boldsymbol{X},t) = \frac{\boldsymbol{j}}{\rho}(\boldsymbol{X},t)$$

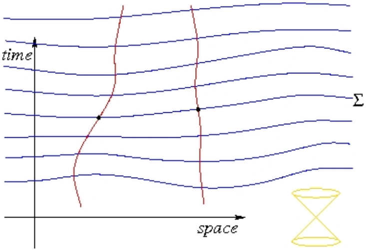
world lines = integral curves of current 4-vector field $j^\mu = \overline{\psi}\gamma^\mu\psi$
world lines are timelike or lightlike at every point

$|\psi|^2$ is conserved in every Lorentz frame.

Consider N particles. Suppose that, for every $\Sigma \in \mathcal{F}$, we have ψ_Σ on Σ^N. $Q(\Sigma) = (Q_1 \cap \Sigma, \ldots, Q_N \cap \Sigma)$ = configuration on Σ

Equation of motion:

$$\frac{dQ_k^\mu}{d\tau} = \mathit{expression}\Big[\psi(Q(\Sigma))\Big]$$



Example for N Dirac particles

$\psi_\Sigma : \Sigma^N \to (\mathbb{C}^4)^{\otimes N}$. Equation of motion:

$$\frac{dQ_i^{\mu_i}(s)}{d\tau} \propto \overline{\psi}(Q(\Sigma))\big[\gamma^{\mu_1} \otimes \cdots \otimes \gamma^{\mu_N}\big]\psi(Q(\Sigma)) \prod_{k \neq i} n_{\mu_k}(Q_k \cap \Sigma)$$

with $n_\mu(x)$ = unit normal vector to Σ at $x \in \Sigma$.

Key facts about BM$_{\mathcal{F}}$

Known in the case of N non-interacting Dirac particles, expected to be true also, say, one day, in full QED:

Equivariance

Suppose initial configuration is $|\psi|^2$-distributed. Then the configuration of crossing points $Q(\Sigma) = (Q_1 \cap \Sigma, \ldots, Q_N \cap \Sigma)$ is $|\psi_\Sigma|^2$-distributed (in the appropriate sense) on every $\Sigma \in \mathcal{F}$.

Predictions

The detected configuration is $|\psi_\Sigma|^2$-distributed on *every* spacelike Σ.
No superluminal signaling. [Lienert and Tumulka 1706.07074]

As a consequence,

$\mathcal{F}$ is invisible, i.e., experimental results reveal no information about $\mathcal{F}$.
(Another limitation to knowledge)

- Although it may seem to go against the spirit of relativity, I take seriously the possibility that our world might have a time foliation.
- However, there do exist relativistic realist theories of quantum mechanics that do not require a time foliation: A relativistic version [Tumulka 2004] of the GRW theory.
- The theory is somewhat more complicated and less natural than Bohmian mechanics.
- The wave function ψ_Σ on the spacelike hypersurface Σ is random and evolves according to a stochastic modification of the Schrödinger equation.

GRW in relativistic space-time

Instantaneous collapse

Everybody's first idea:

If collapse is instantaneous (as opposed to propagating at speed c) then it must violate relativity.

That problem is easily avoided [Aharonov and Albert 1981]

For every spacelike hypersurface Σ there is a wave fct $\psi_\Sigma \in \mathscr{H}_\Sigma$.

E.g., $\mathscr{H}_\Sigma = \mathscr{H}_1^{\otimes N}$, $\mathscr{H}_1 = L^2\Big(\Sigma, \mathbb{C}^4, \langle\phi|\psi\rangle = \int_\Sigma d^3x\, \overline{\phi}(x) n_\mu(x) \gamma^\mu \psi(x)\Big)$.

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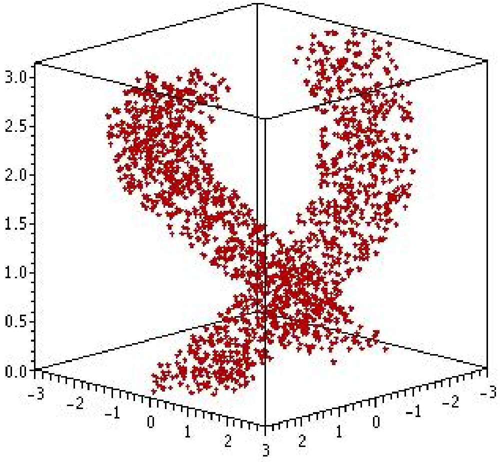
Flash ontology



Flashes in 2+1-dim space-time forming a binary star

Relativistic GRW model

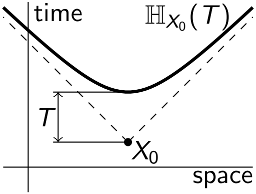
[Tumulka quant-ph/0406094, quant-ph/0602208, 0711.0035, 2002.00482]

- fixed number N of distinguishable particles
- works also in curved space-time, described here in Minkowski space-time $\mathbb{M} = \mathbb{R}^4$
- works also for interacting particles [2002.00482], described here for non-interacting ones
- works also with matter density ontology [Bedingham et al. 1111.1425], described here with flash ontology
- unitary part of evolution is regarded as given: e.g., free Dirac [arising from $L^2(\mathbb{R}^3, \mathbb{C}^4)$]
- with every spacelike surface Σ there is associated a Hilbert space $\mathscr{H}_\Sigma$
- unitary evolution $U_\Sigma^{\Sigma'}$

The rGRW process for $N = 1$

Given: initial wave fct ψ_0 on some 3-surface Σ_0, seed flash $X_0 \in \mathbb{M}$

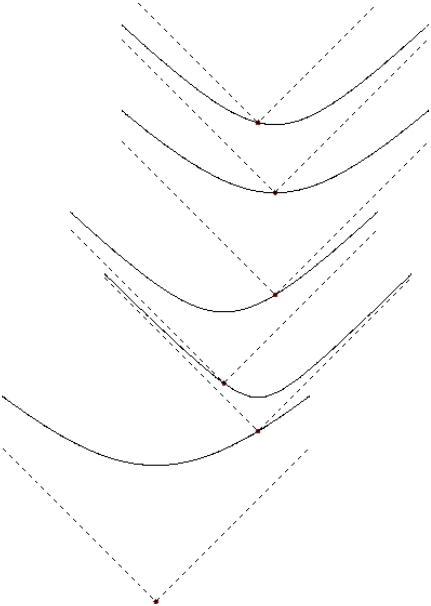
Randomly select next flash $X \in \mathbb{M}$:



- Randomly select waiting time $T \sim \text{Exp}(\lambda)$, $T =$ proper time between X_0 and X, i.e., $X \in \mathbb{H}_{X_0}(T)$
- Evolve $\psi_0 \to \psi_\Sigma$ from Σ_0 to $\Sigma = \mathbb{H}_{X_0}(T)$.
- Randomly select $X \in \Sigma$ with probability density $|\psi_\Sigma|^2 * g$, where $* =$ convolution and g the Gaussian on Σ

$$g(z) = \mathcal{N} \exp\Big(-\frac{\text{dist}_\Sigma(x,z)^2}{2\sigma^2}\Big),$$

$\text{dist}_\Sigma(x,z) =$ spacelike dist. from x to z along Σ, normalization $\int_\Sigma d^3x \, g_x(z) = 1$.



Repeat with

ψ_0 replaced by $\dfrac{g_X \psi_\Sigma}{\|g_X \psi_\Sigma\|}$

and X_0 by X.

The rGRW process for $N = 1$

It follows from the definition that the joint distribution of the first n flashes is of the form

$$\mathbb{P}\Big((X_1, \ldots, X_n) \in B\Big) = \langle \psi_0 | G_{1n}(B) | \psi_0 \rangle, \qquad B \subseteq (\mathbb{R}^4)^n$$

where $\psi_0 \in L^2(\Sigma_0)$, and G_{1n} is a positive-operator-valued measure (POVM). Set $G_1 := \lim_{n\to\infty} G_{1n}$.

The rGRW process for $N > 1$

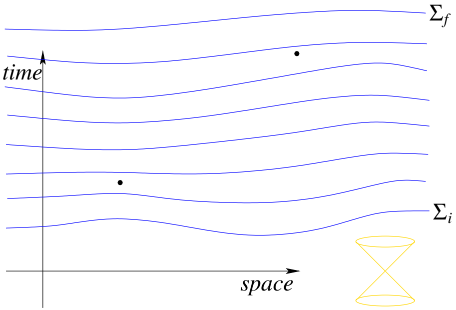
Let the joint probability distribution of the flashes for particles $1 \ldots N$ be

$$\mathbb{P}\Big((X_{11}, X_{12}, \ldots) \in B\Big) = \langle \psi_0 | G_N(B) | \psi_0 \rangle$$

where $\psi_0 \in L^2(\Sigma_0)^{\otimes N}$, and G_N is the product POVM defined by

$$G_N(B_1 \times \cdots \times B_N) = G_1(B_1) \otimes \cdots \otimes G_1(B_N).$$

- We have defined the joint distribution of the flashes.
- random wave function ψ_Σ:
- If the flashes X_{ik} up to Σ are given, ψ_Σ is determined by the initial $\psi_0 \in \mathscr{H}_{\Sigma_0}$: Roughly speaking, collapse ψ at every flash and evolve ψ unitarily in-between.



No signaling

The distribution of the flashes of particle 1 does not depend on the external field A_μ applied to other particles at spacelike separation. It does not depend either on the external field A_μ applied to particle 1 at spacelike separation, except in a neighborhood of size 10^{-7} m and 10^{-8} s.

Nonlocality

The flash process F is nonlocal, i.e., if the space-time regions A and B are spacelike separated then, in general, flashes in A are *not* conditionally independent of those in B, given their common past:

$$\mathbb{P}\Big(F\cap A\Big|F\cap B, F\cap past(A)\cap past(B)\Big) \neq \mathbb{P}\Big(F\cap A\Big|F\cap past(A)\cap past(B)\Big).$$

But there is no fact about who influences whom.

Relativistic GRWf illustrates that a theory can be relativistic and nonlocal.

A few remarks on quantum field theory

Extending Bohmian mechanics to quantum field theory

Two approaches:

1. "Field ontology":
   Instead of an actual configuration $(\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_N)$ of particles, postulate an actual field configuration $\Phi(\boldsymbol{x})$; the quantum state is a wave functional $\Psi[\phi]$ on the ∞-dimensional space of all field configurations $\phi = \phi(\boldsymbol{x})$. Equation of motion

$$\frac{\partial \Phi}{\partial t} = \mathrm{Im}\left[\frac{1}{\Psi[\Phi]}\frac{\delta \Psi}{\delta \phi}\Big|_{\phi=\Phi}\right]$$

2. "Particle ontology":
   Trajectories for photons, electrons, positrons, etc.
   Particles can be created and annihilated.

Particle creation in Bohmian mechanics

[Bell 1986, Dürr, Goldstein, Tumulka, Zanghì quant-ph/0208072]

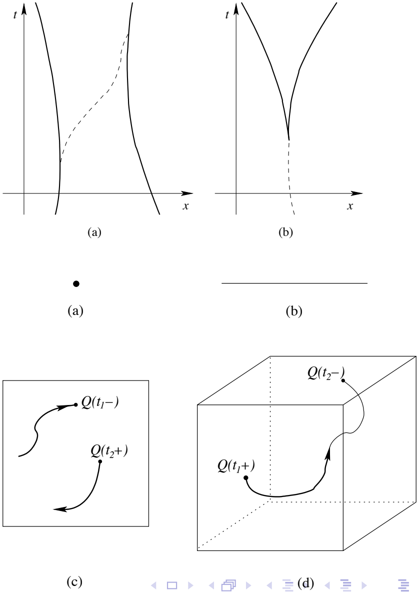
Natural extension of Bohmian mechanics to particle creation:

$$\Psi \in \text{Fock space} = \bigoplus_{N=0}^{\infty} \mathscr{H}_N,$$

configuration space of a variable number of particles

$$= \bigcup_{N=0}^{\infty} \mathbb{R}^{3N}$$

jumps (e.g., n-sector $\to$ $(n+1)$-sector) occur in a stochastic way, with rates governed by a further equation of the theory.



Jump rate formula

- Jump rate from q' to $q \in \mathcal{Q}$:

$$\sigma^\psi(q' \to q) = \frac{\max\{0, \frac{2}{\hbar}\,\mathrm{Im}\,\langle\psi|P(q)H_I P(q')|\psi\rangle\}}{\langle\psi|P(q')\,\psi\rangle}$$

- here, $H_I =$ interaction Hamiltonian, $H = H_0 + H_I$, and
- $P(q)$ the configuration operators
  - e.g., $P(q) = |q\rangle\langle q|$
  - or generally, a POVM (positive-operator-valued measure) on configuration space
- Between jumps, Bohm's equation of motion applies.
- $|\psi|^2$ distribution $\langle\psi|P(q)|\psi\rangle$ holds at every time t.

Essentially, if you have a Hilbert space $\mathscr{H}$, a state vector $\psi \in \mathscr{H}$, a Hamiltonian H, a configuration space $\mathcal{Q}$, and configuration operators $P(q)$, then we know how to set up Bohmian trajectories $Q(t)$.

Thank you for your attention