Bohmian Trajectories as the Foundation of Quantum Mechanics and Quantum Field Theory

Roderich Tumulka

Eberhard Karls University, Tübingen (Germany)

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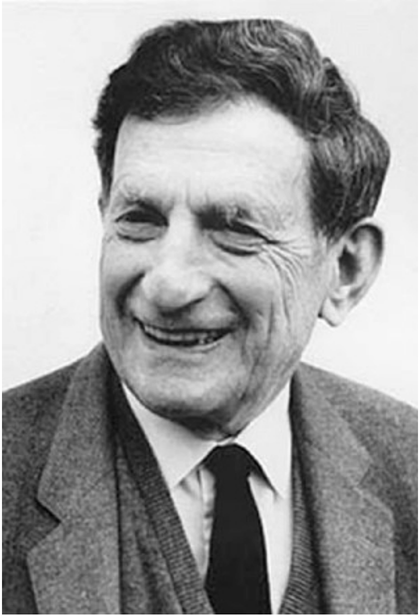


Happy 100-th birthday, David Bohm!

- In 1952, David Bohm solved the biggest of all problems in quantum mechanics,
- which is to provide an explanation of quantum mechanics.
- His theory is known as Bohmian mechanics, pilot-wave theory, de Broglie–Bohm theory, or the ontological interpretation.
- This theory makes a proposal for how the our world might work.
- It agrees with all empirical observations of quantum mechanics.
- It is widely under-appreciated.
- It achieves what was often (before and even after 1952) claimed impossible: To explain the rules of quantum mechanics through a coherent picture of microscopic reality.
- It is remarkably simple and elegant.
- It is probably the true theory of quantum reality.

- Compared to Bohmian mechanics, orthodox quantum mechanics appears quite "unprofessional" (John Bell) and "incoherent" (Albert Einstein).
- In fact, orthodox quantum mechanics appears like the narrative of a dream whose logic doesn't make sense any more once you are awake although it seemed completely natural while you were dreaming.
- According to Bohmian mechanics, electrons and other elementary particles are particles in the literal sense, i.e., they have a well-defined position $\boldsymbol{Q}_j(t) \in \mathbb{R}^3$ at all times t. They have trajectories.
- These trajectories are governed by Bohm's equation of motion (next slide).
- Given the claim that it was impossible to explain quantum mechanics, it is remarkable that something as simple as particle trajectories does the job.
- What went wrong in orthodox QM? Some variables were left out of consideration: the particle positions!

Laws of Bohmian mechanics

1. Bohm's equation of motion

$$\frac{d\boldsymbol{Q}_j}{dt} = \frac{\hbar}{m_j} \mathrm{Im} \frac{\psi^* \nabla_j \psi}{\psi^* \psi}(\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_N)$$

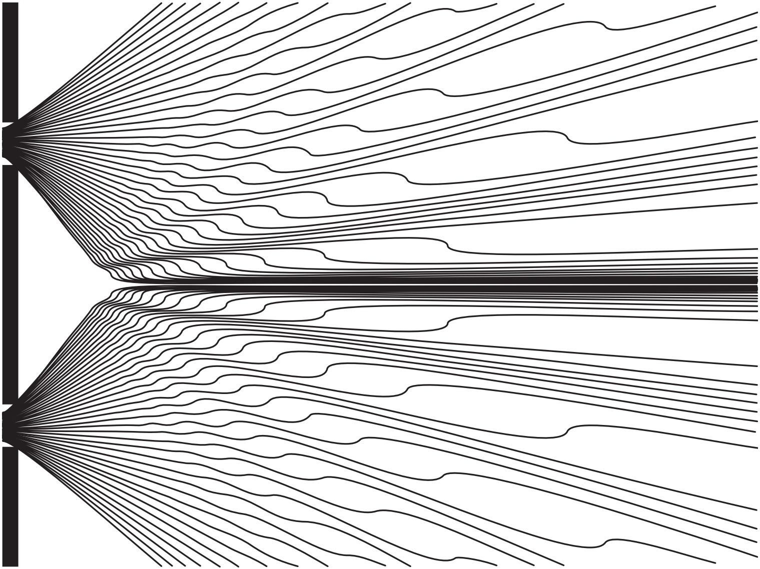
2. The Schrödinger equation for ψ,

$$i\hbar \frac{\partial \psi}{\partial t} = -\sum_j \frac{\hbar^2}{2m_j} \nabla_j^2 \psi + V\psi$$

3. The initial configuration $Q(0) = (\boldsymbol{Q}_1(0), \ldots, \boldsymbol{Q}_N(0))$ is random with probability density

$$\rho = |\psi_0|^2 \,.$$

It follows that at any time $t \in \mathbb{R}$, $Q(t)$ is random with density $\rho_t = |\psi_t|^2$ ("equivariance theorem").



Drawn by G. Bauer after Philippidis et al. [1979]

"This idea seems to me so natural and simple, to resolve the wave–particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored."

John S. Bell (1928–1990)

- Bohmian mechanics is clearly non-local.
- Bohmian mechanics avoids the problematical idea that the world consists only of wave function.
- It provides precision, clarity, and a clear ontology in space-time.
- It allows for an analysis of quantum measurements, thus replacing postulates of orthodox QM by theorems.

Extensions of Bohmian mechanics

- Particle creation
- Relativistic space-time

Particle creation in Bohmian mechanics

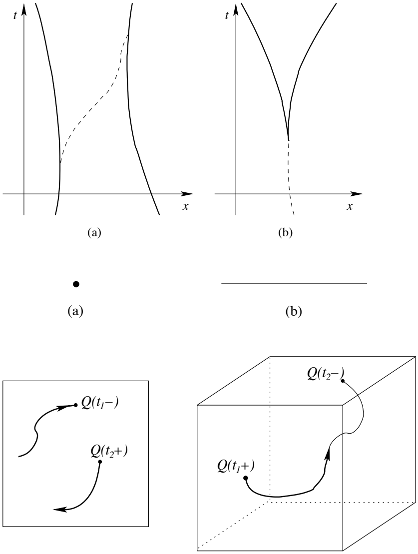
[Bell 1986; Dürr, Goldstein, Tumulka, Zanghì 2005]

Natural extension of Bohmian mechanics to particle creation:

$\Psi \in$ Fock space $\mathscr{F} = \bigoplus_{n=0}^{\infty} \mathscr{H}_n$,

configuration space of a variable number of particles
$\mathcal{Q} = \bigcup_{n=0}^{\infty} \mathbb{R}^{3n}$

jumps (e.g., n-sector $\to$ $(n+1)$-sector) occur in a stochastic way, with rates governed by a further equation of the theory.



(c) (d)

Jump rate formula

- Jump rate from q' to $q \in \mathcal{Q}$:

$$\sigma^\psi(q' \to q) = \frac{\max\left\{0, \frac{2}{\hbar}\,\mathrm{Im}\,\langle\psi|P(q)H_I P(q')|\psi\rangle\right\}}{\langle\psi|P(q')\,\psi\rangle}$$

- here, H_I = interaction Hamiltonian, $H = H_0 + H_I$, and
- $P(q)$ the configuration operators
  - e.g., $P(q) = |q\rangle\langle q|$
  - or generally, a POVM (positive-operator-valued measure) on configuration space
- Between jumps, Bohm's equation of motion applies.
- $|\psi|^2$ distribution $= \langle\psi|P(q)|\psi\rangle$ holds at every time t.

Essentially, if you have a Hilbert space $\mathscr{H}$, a state vector $\psi \in \mathscr{H}$, a Hamiltonian H, a configuration space $\mathcal{Q}$, and configuration operators $P(q)$, then we know how to set up Bohmian trajectories $Q(t)$.

An UV divergence problem

For example, consider a simplified model QFT:

- x-particles can emit and absorb y-particles.
- There is only 1 x-particle, and it is fixed at the origin. $\mathscr{H} = \mathscr{F}_y^{\text{bosonic}}$
- configuration space $\mathcal{Q} = \bigcup_{n=0}^{\infty} \mathbb{R}^{3n}$

Original Hamiltonian in the particle-position representation:

$$(H_{\text{orig}}\psi)(\boldsymbol{y}_1 \ldots \boldsymbol{y}_n) = -\frac{\hbar^2}{2m_y}\sum_{j=1}^{n} \nabla^2_{\boldsymbol{y}_j}\psi(\boldsymbol{y}_1 \ldots \boldsymbol{y}_n)$$
$$+ \; g\sqrt{n+1}\,\psi(\boldsymbol{y}_1 \ldots \boldsymbol{y}_n, \boldsymbol{0})$$
$$+ \; \frac{g}{\sqrt{n}} \sum_{j=1}^{n} \delta^3(\boldsymbol{y}_j)\,\psi(\boldsymbol{y}_1 \ldots \widehat{\boldsymbol{y}_j} \ldots \boldsymbol{y}_n)\,,$$

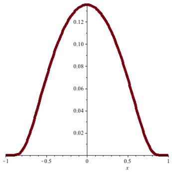
is UV divergent. ($\widehat{\ } =$ omit)

Well-defined, regularized version of H

UV cut-off $\varphi \in L^2(\mathbb{R}^3)$:

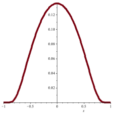
$$(H_{\mathrm{cutoff}}\psi)(\boldsymbol{y}_1 \ldots \boldsymbol{y}_n) = -\frac{\hbar^2}{2m_y}\sum_{j=1}^{n} \nabla^2_{\boldsymbol{y}_j}\psi(\boldsymbol{y}_1 \ldots \boldsymbol{y}_n) + $$
$$+ \; g\sqrt{n+1}\sum_{i=1}^{m}\int_{\mathbb{R}^3} d^3\boldsymbol{y}\,\varphi^*(\boldsymbol{y})\,\psi\big(\boldsymbol{y}_1 \ldots \boldsymbol{y}_n, \boldsymbol{y}\big) +$$
$$+ \; \frac{g}{\sqrt{n}}\sum_{i=1}^{m}\sum_{j=1}^{n}\varphi(\boldsymbol{y}_j)\,\psi\big(\boldsymbol{y}_1 \ldots \widehat{\boldsymbol{y}}_j \ldots \boldsymbol{y}_n\big)$$

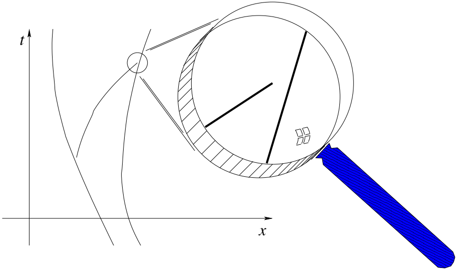
"smearing out" the x-particle
with "charge distribution" $\varphi(\cdot)$



But then . . .

. . . emission and absorption occurs anywhere in a ball around the x-particle (= in the support of

$\varphi =$ )

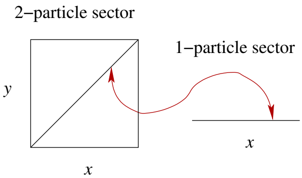


This UV problem can be solved!

[Teufel and Tumulka 2015; Lampart, Schmidt, Teufel, and Tumulka 2017]

Novel idea: Interior-boundary condition

Here: boundary config = where y-particle meets x-particle;
interior config = one y-particle removed



Interior–boundary condition (IBC)

$$\psi^{(n+1)}(\mathrm{bdy}) = (\mathrm{const.})\,\psi^{(n)}$$

links two configurations connected by the creation or annihilation of a particle.
For example, with an x-particle at $\mathbf{0}$,

$$\psi(y^n, \mathbf{0}) = \frac{g\, m_y}{2\pi\hbar^2\sqrt{n+1}}\,\psi(y^n)\,.$$

with $y^n = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_n)$.

Self-adjoint Hamiltonian, rigorously

- IBC $\lim_{r\searrow 0} r\psi(y^n, r\boldsymbol{\omega}) = \dfrac{g\,m_y}{2\pi\hbar^2\sqrt{n+1}}\,\psi(y^n)$ (1)

- $$H_{IBC}\psi = H_{\text{free}}\psi + \frac{g\sqrt{n+1}}{4\pi}\int_{\mathbb{S}^2} d^2\boldsymbol{\omega}\,\lim_{r\searrow 0}\frac{\partial}{\partial r}\Big(r\psi(y^n, r\boldsymbol{\omega})\Big)$$
$$+\frac{g}{\sqrt{n}}\sum_{j=1}^{n}\delta^3(\boldsymbol{y}_j)\,\psi(y^n\setminus\boldsymbol{y}_j) \qquad (2)$$

Theorem [Lampart, Schmidt, Teufel, Tumulka 2017]

On a suitable dense domain $\mathscr{D}_{IBC}$ of ψs in $\mathscr{H}$ satisfying the IBC (1), H_{IBC} is well defined, self-adjoint, and positive. No UV divergence!

Bohmian particles:

- when $Q(t)\in\mathcal{Q}^{(n)}$ reaches $\boldsymbol{y}_j=\mathbf{0}$, it jumps to $(y^n\setminus\boldsymbol{y}_j)\in\mathcal{Q}^{(n-1)}$
- emission of new y-particle at $\mathbf{0}$ at random time with random direction with a rate dictated by time reversal invariance.

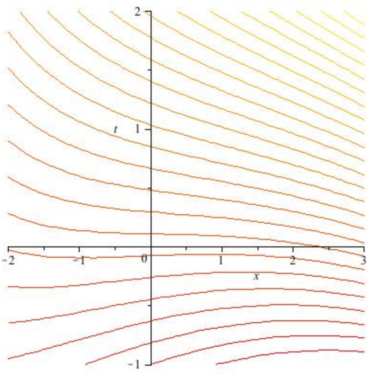
Extensions of Bohmian mechanics: Relativistic space-time

Bohmian mechanics in relativistic space-time

- If a preferred foliation (= slicing) of space-time into spacelike hypersurfaces ("time foliation" $\mathcal{F}$) is permitted, then there is a simple, convincing analog of Bohmian mechanics, $\mathrm{BM}_{\mathcal{F}}$.

  [Bohm and Hiley 1993 for flat foliations;
  Dürr, Goldstein, Münch-Berndl, Zanghì 1999 for curved foliations;
  Tumulka 2001 for curved space-time]

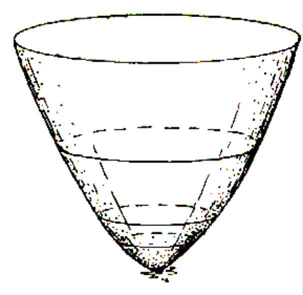
- Without a time foliation, no version of Bohmian mechanics is known that would make predictions anywhere near quantum mechanics. (And I have no hope that such a version can be found in the future.)



- To grant a time foliation (= preferred foliation) is against the spirit of relativity.
- But it is a real possibility that our world is like that.
- It doesn't mean relativity would be irrelevant:
  - There is still a metric $g_{\mu\nu}$.
  - The free Hamiltonian is still the Dirac operator.
  - Formulas are still expressed with 4-vector indices (j^μ etc.),
  - Just there is also the vector n_μ normal to the time foliation.
- Still no superluminal signaling.
- The hypothesis of a time foliation provides a very simple explanation of the non-locality required by Bell's theorem.

A preferred foliation may be provided anyhow by the metric:

Simplest choice of time foliation $\mathcal{F}$



Drawing: R. Penrose

Let $\mathcal{F}$ be the level sets of the function T : space-time $\to \mathbb{R}$,
$T(x) =$ timelike-distance$(x,$ big bang$)$.

E.g., $T(\text{here-now}) = 13.7$ billion years

Alternatively, $\mathcal{F}$ might be defined in terms of the quantum state vector ψ, $\mathcal{F} = \mathcal{F}(\psi)$ [Dürr, Goldstein, Norsen, Struyve, Zanghì 2014]

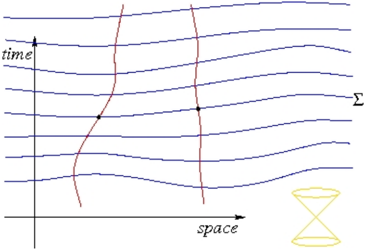
Or, $\mathcal{F}$ might be determined by an evolution law (possibly involving ψ) from an initial time leaf.

$\mathrm{BM}_{\mathcal{F}}$ [Dürr et al. 1999]

Consider N particles. Suppose that, for every $\Sigma \in \mathcal{F}$, we have ψ_Σ on Σ^N. $Q(\Sigma) = (Q_1 \cap \Sigma, \ldots, Q_N \cap \Sigma)$ = configuration on Σ

Equation of motion:

$$\frac{dQ_k^\mu}{d\tau} = \mathit{expression}\Big[\psi(Q(\Sigma))\Big]$$



Example for N Dirac particles

$\psi_\Sigma : \Sigma^N \to (\mathbb{C}^4)^{\otimes N}$. Equation of motion:

$$\frac{dQ_k^\mu}{d\tau} \propto j_k^\mu(Q(\Sigma)),$$

$$j^{\mu_1 \cdots \mu_N} = \overline{\psi}[\gamma^{\mu_1} \otimes \cdots \otimes \gamma^{\mu_N}]\psi,$$

$$j_k^{\mu_k}(q_1 \ldots q_N) = j^{\mu_1 \cdots \mu_N}(q_1 \ldots q_N)\, n_{\mu_1}(q_1) \cdots (k\text{-th omitted}) \cdots n_{\mu_N}(q_N)$$

with $n_\mu(x)$ = unit normal vector to Σ at $x \in \Sigma$.

Key facts about $\mathrm{BM}_{\mathcal{F}}$

Equivariance

Suppose initial configuration is $|\psi|^2$-distributed. Then the configuration of crossing points $Q(\Sigma) = (Q_1 \cap \Sigma, \ldots, Q_N \cap \Sigma)$ is $|\psi_\Sigma|^2$-distributed (in the appropriate sense) on every $\Sigma \in \mathcal{F}$.

Predictions

The detected configuration is $|\psi_\Sigma|^2$-distributed on *every* spacelike Σ.

As a consequence,

$\mathcal{F}$ is invisible, i.e., experimental results reveal no information about $\mathcal{F}$.

All empirical predictions of $\mathrm{BM}_{\mathcal{F}}$

agree with the standard quantum formalism and the empirical facts.

Key facts about BM$_{\mathcal{F}}$

Theorem [Lienert and Tumulka 2017]

If detectors are placed along any spacelike surface Σ, the joint distribution of detection events is $|\psi_\Sigma|^2$.

BM$_{\mathcal{F}}$ is very robust:

- works for arbitrary foliation $\mathcal{F}$
- works even if the foliation has kinks [Struyve and Tumulka 2014]
- works even if the leaves of $\mathcal{F}$ overlap [Struyve and Tumulka 2015]
- can be combined with the stochastic jumps for particle creation
- works also in curved space-time [Tumulka 2001]
- works still if space-time has singularities [Tumulka 2010]

Multi-time wave function $\phi(t_1, x_1, \ldots, t_N, x_N)$

as a generalization of the N-particle wave function $\psi(t, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_N)$ of non-relativistic quantum mechanics:

$$\psi(t, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_N) = \phi(t, \boldsymbol{x}_1, \ldots, t, \boldsymbol{x}_N)$$

$\psi_\Sigma(x_1, \ldots, x_N) = \phi(x_1, \ldots, x_N)$.
Intended: if detectors along Σ then prob distribution of outcomes $= |\psi_\Sigma|^2$

$$i\frac{\partial \psi}{\partial t} = H\psi \quad \text{vs.} \quad i\frac{\partial \phi}{\partial t_i} = H_i\phi \quad \forall i = 1, \ldots, N$$

$$\sum_{i=1}^{N} H_i = H$$

It's the covariant particle-position representation of the state vector.
Closely related to the Tomonaga-Schwinger wave function, but simpler.

Multi-time wave function $\phi(t_1, x_1, \ldots, t_N, x_N)$

Consistency condition

$$\left[i\frac{\partial}{\partial t_i} - H_i, i\frac{\partial}{\partial t_j} - H_j \right] = 0 \quad \forall i \neq j$$

[Dirac, Fock, Podolsky 1932; F. Bloch 1934]

- trivially satisfied for non-interacting particles
- interaction is a challenge, potentials violate consistency
- zero-range interactions possess consistent multi-time equations [Lienert 2015]
- interaction through emission and absorption of bosons possesses consistent multi-time equations [Petrat and Tumulka 2014]

Thank you for your attention