On the Present Status of Quantum Mechanics

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  where $m_k > 0, e_k \in \mathbb{R}, G > 0$ are constants.
- This theory is called Newtonian mechanics,
  the first sum the "gravitational force,"
  the second the "Coulomb force,"
  m_k the mass of particle k, and e_k its charge.

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where $S : (\mathbb{E}^3)^N \times \mathbb{E}^1 \to \mathbb{R}$ is called the Hamilton–Jacobi function.

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$$i\hbar \frac{\partial \psi}{\partial t}(\boldsymbol{q}_1 ... \boldsymbol{q}_N, t) = -\sum_{k=1}^{N} \frac{\hbar^2}{2m_k} \nabla^2_{\boldsymbol{q}_k} \psi + \sum_{j \neq k} \frac{e_j e_k}{|\boldsymbol{q}_j - \boldsymbol{q}_k|} \psi\,,$$

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One more axiom of Bohmian mechanics

We write $Q(t) := (\boldsymbol{Q}_1(t), \ldots, \boldsymbol{Q}_N(t)) =:$ configuration at time t

Axiom

At the initial time $t = 0$ of the universe, $Q(0)$ is random with probability density $\rho(Q(0) = q) = |\psi(q, t = 0)|^2$. In short, $Q(0) \sim |\psi_0|^2$.

In particular, assume $\psi_0 := \psi(\cdot, t = 0) \in L^2(\mathbb{R}^{3N}, \mathbb{C})$ with $\|\psi_0\|^2 = 1$.

Existence theorem [Dürr et al. quant-ph/9503013, Teufel and Tumulka math-ph/0406030]

For large classes of initial wave functions ψ_0 and potentials V, the solution $t \mapsto Q(t)$ of Bohm's equation of motion almost surely exists for all times.

Equivariance theorem

$Q(t) \sim |\psi_t|^2$ for all t.

The key fact about Bohmian mechanics

As a consequence of the definition of the theory:

Observers inhabiting a Bohmian universe (made out of Bohmian particles) observe random-looking outcomes of their experiments whose statistics agree with the rules of quantum mechanics for making predictions.

A theory like this was believed to be impossible

Werner Heisenberg in 1958:

"We can no longer speak of the behavior of the particle independently of the process of observation."
"The idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them [...], is impossible."

Heisenberg was wrong. Bohmian mechanics is a counter-example to the impossibility claim.

W. Heisenberg (1901–1976)

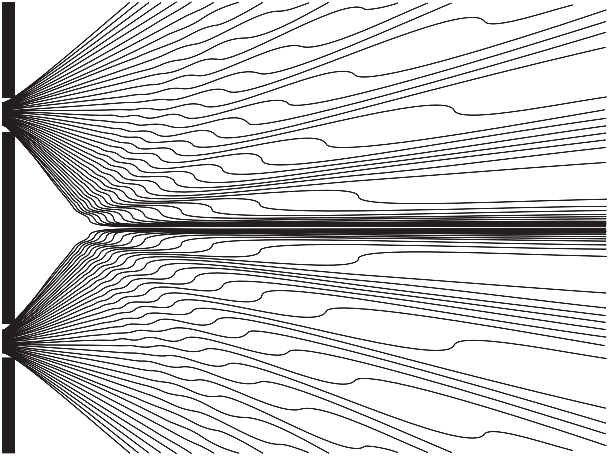
Rules of QM

Nearly all views about QM agree about the rules for making empirical predictions:

- Unitary evolution: The wave function φ of an isolated system evolves according to the Schrödinger equation, $\varphi_t = e^{-iHt/\hbar}\varphi_0$ with H the Hamiltonian operator in Hilbert space $\mathscr{H}$ such as $L^2(\mathbb{R}^{3M}, \mathbb{C})$.
- Born's rule: When an observer makes a "quantum measurement" of the observable $\mathscr{A}$ associated with the self-adjoint operator A with spectral decomposition $A = \sum_\alpha \alpha P_\alpha$ on a system with wave function φ, the outcome is the eigenvalue α with probability $\|P_\alpha \psi\|^2 = \langle \psi | P_\alpha \psi \rangle$.
- Collapse rule: After a quantum measurement of $\mathscr{A}$ with outcome α, the wave function gets replaced by

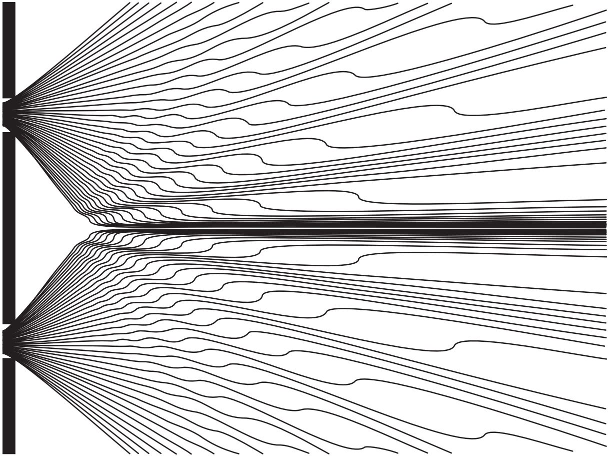
$$\psi_{t+} = \frac{P_\alpha \psi_{t-}}{\|P_\alpha \psi_{t-}\|}\,.$$

Example: the double-slit experiment



Drawn by G. Bauer after Philippidis et al.

Shown: A double-slit and 80 possible paths of Bohm's particle. The wave passes through both slits, the particle through only one.



Most paths arrive where $|\psi|^2$ is large—that's how the interference pattern arises. If one slit gets closed, the wave passes through only one slit, which leads to different trajectories and no interference pattern. Bohmian mechanics takes wave–particle dualism literally: there is a wave, and there is a particle. The path of the particle depends on the wave.

Limitations to Knowledge

Limitations to Knowledge (1)

Definition

For a system with configuration $X \in \mathbb{R}^{3M}$, $Q = (X, Y)$, the conditional wave function φ is defined to be $\varphi(x, t) = \mathcal{N}\, \psi(x, Y(t), t)$ with normalizing factor $\mathcal{N}$.

Theorem

Inhabitants of a Bohmian universe cannot know the configuration X of a system more precisely than that it is $|\varphi(x)|^2$ distributed.

Sketch of proof: Let Y = rest of the universe; knowledge is encoded in Y. For any set A, $\mathbb{P}(X \in A | Y) = \mathcal{N}^2 \int_A dx\, |\psi(x, Y)|^2 = \int_A dx\, |\varphi(x)|^2$. $\square$

Limitations to Knowledge (2)

In any version of QM, you cannot measure the wave function.

- Example: Alice chooses a direction $\boldsymbol{a}$ in space and prepares a spin-$\frac{1}{2}$ particle with $\psi = |\text{spin up in } \boldsymbol{a}\rangle$.
- She hands it to Bob with the challenge to determine ψ (or $\boldsymbol{a}$).
- According to the rules of QM, Bob can do no better than perform a Stern-Gerlach experiment in a direction $\boldsymbol{b}$ of his choice and obtain 1 bit ("up" or "down").
- He can conclude whether $\boldsymbol{a}$ is more likely to lie in the hemisphere closer to $\boldsymbol{b}$ or closer to $-\boldsymbol{b}$, but cannot determine $\boldsymbol{a}$.
- (If the game is repeated and Alice always prepares the same ψ, Bob can determine ψ to desired accuracy. But not in a single run.)
- Nature knows in every single run what ψ is because Alice knows, and she can prove it. So,

Limitation to knowledge

Certain variables have well-defined values in the world (known to nature), although we cannot measure them, even with all future advances.

Limitations to Knowledge (3)

Limitations to knowledge may seem to conflict with some principle of science, such as

> *"a statement is unscientific or even meaningless if it cannot be tested experimentally, an object is not real if it cannot be observed, and a variable is not well-defined if it cannot be measured."*

- But limitations to knowledge are a fact of quantum mechanics.
- Get used to them!
- The "principle" above is not a principle at all, it is wrong. It is exaggerated positivism.

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It was long thought that the key to clarity in QM was to avoid talking about ontology and stick to operational statements. That thought has not paid off.

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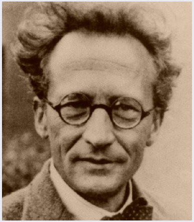
  where ψ evolves according to the Schrödinger equation.
- This is called Schrödinger's first theory.

$$m(\boldsymbol{x},t) = \sum_{k=1}^{N} \int_{(\mathbb{E}^3)^N} dq\, \delta^3(\boldsymbol{x} - \boldsymbol{q}_k)\, |\psi(q,t)|^2$$

He soon abandoned this theory because he thought it made wrong predictions. But actually, it makes correct predictions, but it has a many-worlds character.



E. Schrödinger (1887–1961)

For Schrödinger's cat, $\psi = \frac{1}{\sqrt{2}}\psi_{\text{dead}} + \frac{1}{\sqrt{2}}\psi_{\text{alive}}$, it follows that $m = \frac{1}{2} m_{\mathrm{dead}} + \frac{1}{2} m_{\mathrm{alive}}$.

There is a dead cat and a live cat, but they are like ghosts to each other (they do not notice each other), as they do not interact. So to speak, they live in parallel worlds. [Allori et al. arXiv:0903.2211]

Many worlds

Not knowing about Schrödinger's proposal, Everett advocated a many-worlds view in 1957, but with an inadequate ontology: His idea was that for wave functions such as Schrödinger's cat's, both cats are in the wave function, so both cats exist.

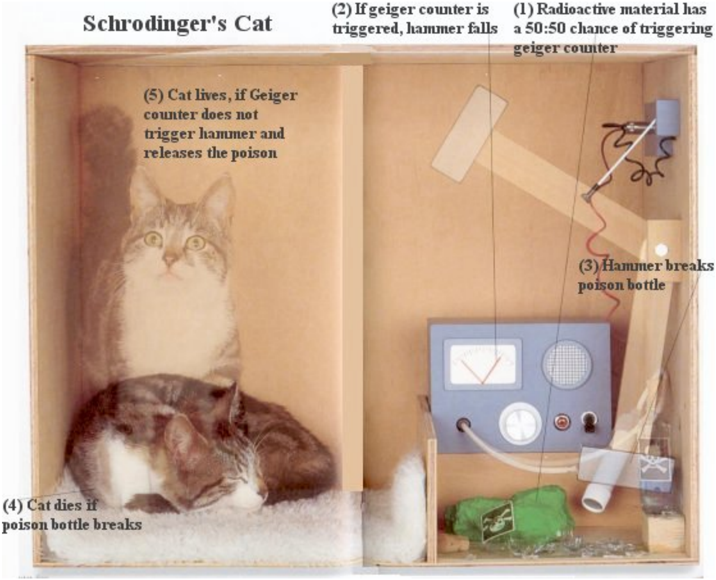
Everett contributed substantially to the analysis of probabilities in a many-world framework.

Hugh Everett (1930–1982)

- Upshot: While Everett's many-worlds theory is ontologically not OK, Schrödinger's is OK.
- Both theories imply that the statistics predicted by the rules of quantum mechanics will be observed in most worlds (i.e., they make correct predictions), provided that we count worlds with $|\psi|^2$ weights.
- It remains philosophically questionable whether a theory with many-worlds characted can postulate weights for counting worlds. That is the real problem about many-worlds.

The Measurement Problem

[E. Schrödinger: Die gegenwärtige Situation in der Quantenmechanik. *Naturwissenschaften* **23**: 807–812, 823–828, 844–849 (1935)]


Schrodinger's Cat
(2) If geiger counter is triggered, hammer falls
(1) Radioactive material has a 50:50 chance of triggering geiger counter
(5) Cat lives, if Geiger counter does not trigger hammer and releases the poison
(3) Hammer breaks poison bottle
(4) Cat dies if poison bottle breaks

What does the cat example mean?

- It's often called a "paradox," but that is too weak—that sounds like "get used to it."
- Basically, it's an argument: Cat + atom belong to a quantum system of 10^{25} electrons, protons and neutrons, with a wave function ψ governed by the Schrödinger equation.
  Since the Schrödinger equation is linear, we have that, after 1 hour, the wave function is a "superposition" of the wave function of a dead cat and that of a live cat:

$$\psi = \psi_{\text{dead}} + \psi_{\text{alive}} \,.$$

  However, in reality the cat must be either dead or alive.

John S. Bell: "The problem is: AND is not OR."

Also known as "the measurement problem of quantum mechanics."

Measurement problem

Consider a quantum measurement of the observable $A = \sum_n \alpha_n |n\rangle\langle n|$.

$$|n\rangle \otimes \phi_0 \xrightarrow{t} |n\rangle \otimes \phi_n$$

($\phi_0 =$ ready state of apparatus, $\phi_n =$ state displaying result α_n)

$$\Rightarrow \quad \sum_n c_n |n\rangle \otimes \phi_0 \xrightarrow{t} \sum_n c_n |n\rangle \otimes \phi_n$$

But one would believe that a measurement has an actual, random outcome n_0, so that one can ascribe the "collapsed state" $|n_0\rangle$ to the system and the state ϕ_{n_0} to the apparatus.

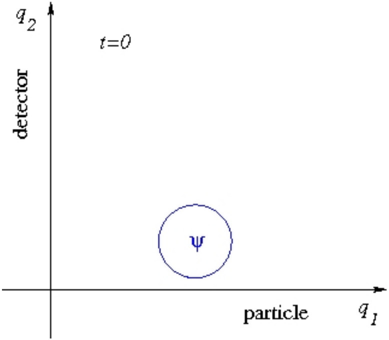
Conclusion from this argument:

- Either ψ is not the complete description of the system,
- or the Schrödinger equation is not correct for $N > 10^{20}$ particles,
- or there are many worlds.

Collapse of the wave function in Bohmian mechanics

The wave function of system and apparatus together does not collapse (but evolves according to the Schrödinger equation).
However, some parts of the wave function become irrelevant to the particles and can be deleted because of decoherence: Two packets of ψ do not overlap in configuration space and will not overlap any more in the future (for the next 10^{100} years).

Evolution of ψ in configuration space of particle + detector:



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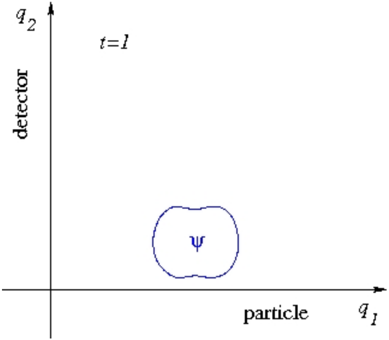

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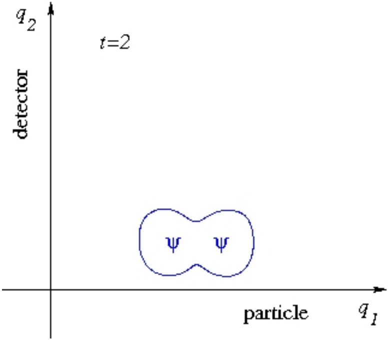

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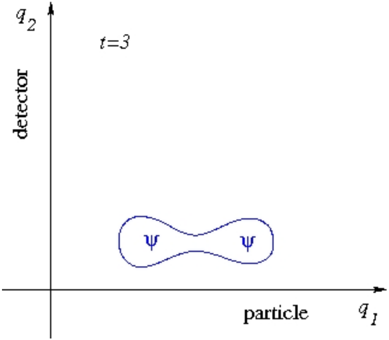

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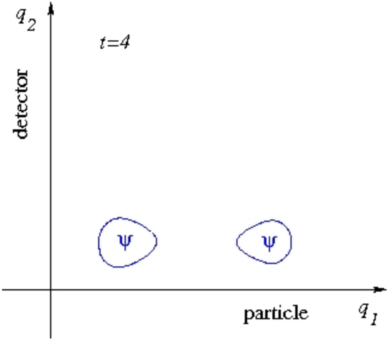

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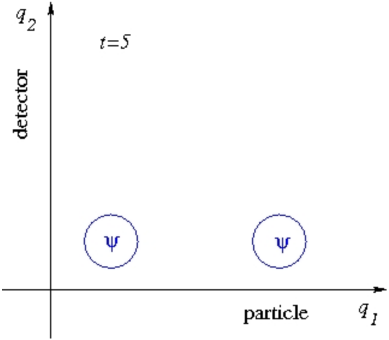

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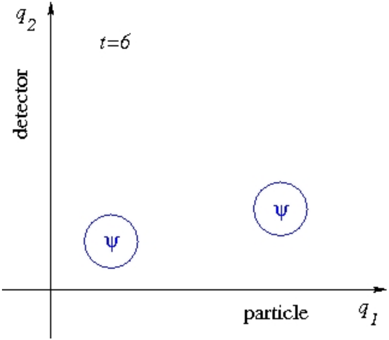

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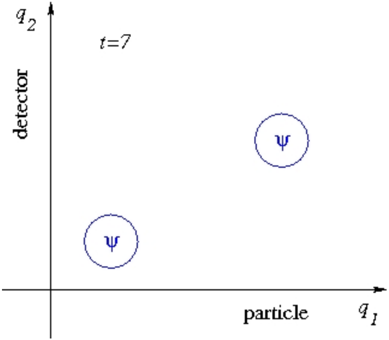

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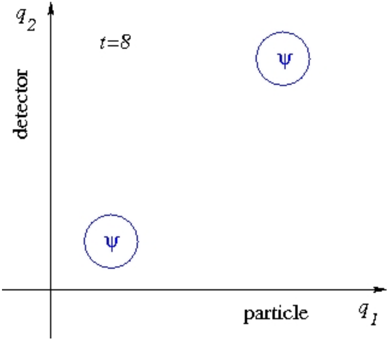

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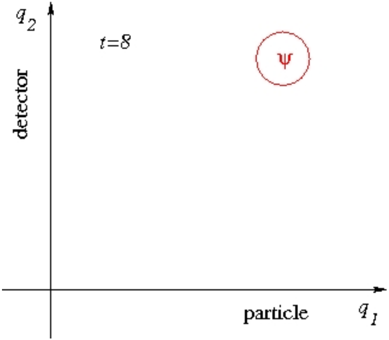

Collapse of the wave function in Bohmian mechanics

- If two packets of ψ do not overlap in configuration space and will not overlap any more in the future (for the next 10^{100} years), then only the packet containing Q will be relevant to the motion of Q (for the next 10^{100} years.
- So the other packets can safely be ignored from now on (although strictly speaking, they still exist) $\Rightarrow$ collapse of ψ
- Probability that ψ collapses to this packet $=$

  probability that Q lies in this packet $=$

  $\int_{\text{packet}} |\psi|^2 = ||\text{packet}||^2$
- Thus, the standard collapse rule comes out.

Thank you for your attention