GRW theory, a spontaneous collapse theory

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- Quantum physics is intriguing.
- It's also mysterious.
- Its predictions agree very precisely with observations.
- But it is impossible to understand.
- Or is it?

- I will describe a theory, GRW theory, that can be completely understood and makes predictions in agreement with quantum mechanics. For all we know, our universe might be governed by GRW theory.
- Do we want to understand QM?
- Even if we don't understand, QM provides rules for calculating predictions for (more or less) any experiment (at low energies, non-relativistic).
- Niels Bohr recommended we don't try to understand. That's the spirit of his "Copenhagen interpretation."
- Ancient astronomers could predict the motion of the planets but didn't understand why planets are moving and stars are fixed. Would you rather be an ancient or a modern astronomer?
- To begin to understand QM, we need to make explicit where the problems with orthodox QM lie.

The quantum measurement problem

What the problem is

- Consider a quantum measurement of some observable $A = \sum \alpha |\alpha\rangle\langle\alpha|$ on a system with wave function $\psi = \sum c_\alpha |\alpha\rangle$.
- The apparatus consists of electrons and quarks, so it should be possible to treat it like a quantum system with a wave fct ϕ on $\mathbb{R}^{3N}$, $N > 10^{23}$.
- If we do, then $\Psi(t_0) = \psi \otimes \phi$ evolves according to the Schrödinger eq. to $\Psi(t_1) = \sum_\alpha c_\alpha \Psi_\alpha$, where Ψ_α corresponds to a needle pointing to α. A superposition of different outcomes.
- $\Psi(t_1)$ doesn't say what the actual outcome is.
- We might have expected a state $\Psi(t_1)$ with a unique needle position.
- We might have expected a random state because the outcome should be random.

Schrödinger's cat

is a particular version of the problem. Schrödinger formulated it to criticize the Copenhagen interpretation of QM.

What exactly is the problem?

Bob: Superposition or not, $\Psi(t_1)$ still yields the right probabilities.

Alice: Everybody agrees about the empirically right probabilities. That is not the problem.

Bob: Then where is the problem?

Alice: The problem is about what is there in reality.

Bob: I believe there is no microscopic reality, that observables don't have values before the measurement.

Alice: But that is a hypothesis about reality, too. If only the wave function exists, that is a hypothesis about reality, too. The measurement problem puts constraints on the possible hypotheses about reality.

Bob: Which constraints?

Let's pin down the problem

3 assumptions

1. In each run of the experiment, there is a unique outcome.
2. The wave function is a complete description of a system's physical state in reality. (There are no further variables.)
3. The time evolution of the wave function of an isolated system, not entangled with the outside, is always given by the Schrödinger eq.

Together, they lead to a contradiction: By 3, $\Psi(t_1)$ is generically a superposition of Ψ_α corresponding to different outcomes. Thus, $\Psi(t_1)$ doesn't select an outcome. If there were further variables (such as Bohm's Q), they could select an outcome, but by 2 there aren't. Thus, there is no unique outcome, in contradiction to 1.

Consequence

We need to drop one of the 3 assumptions.

Bohmian mechanics drops 2, the GRW collapse theory 3, many-worlds 1.

The GRW theory of spontaneous collapse

Spontaneous collapse: GRW theory

Key idea:

The Schrödinger equation is only an approximation, valid for systems with few particles ($N < 10^4$) but not for macroscopic systems ($N > 10^{23}$). The true evolution law for the wave function is non-linear and stochastic (i.e., inherently random) and avoids superpositions (such as Schrödinger's cat) of macroscopically different contributions.

Put differently, regard the collapse of ψ as a physical process governed by mathematical laws.

GianCarlo Ghirardi (1935–2018)

Explicit equations by Ghirardi, Rimini, and Weber [Phys.Rev. D 1986]

The predictions of the GRW theory deviate very very slightly from the quantum formalism. At present, no experimental test is possible.

GRW's stochastic evolution for ψ

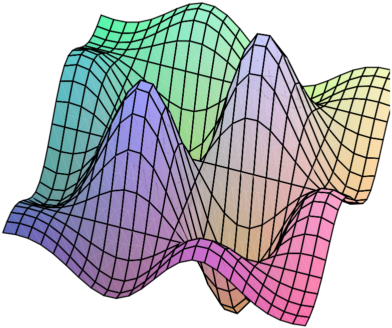
- is designed for non-relativistic quantum mechanics of N particles
- meant to replace Schrödinger eq as a fundamental law of nature
- involves two new constants of nature:
  - $\lambda \approx 10^{-16}\,\text{sec}^{-1}$, called collapse rate per particle.
  - $\sigma \approx 10^{-7}\,\text{m}$, called collapse width.
- Def: ψ evolves as if an observer outside the universe made, at random times with rate $N\lambda$, quantum measurements of the position observable of a randomly selected particle with inaccuracy σ.
- "rate $N\lambda$" means that prob(an event in the next dt seconds) $= N\lambda\, dt$. Waiting time $\sim$ Exp($N\lambda$).
- more explicitly: Schrödinger evolution interrupted by jumps of the form

$$\psi_{T+} = e^{-\frac{(\boldsymbol{q}_k - \boldsymbol{q})^2}{4\sigma^2}} \psi_{T-}\,,$$

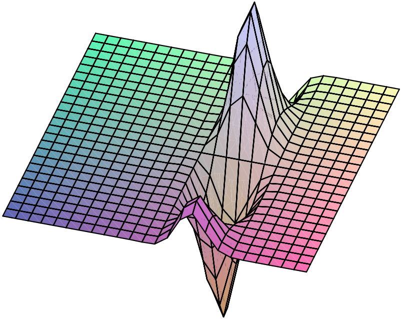
  i.e., multiplication by a Gauss function with random label k, center $\boldsymbol{q}$ and time T.
- prob$(\boldsymbol{q} \in d^3\boldsymbol{q}) = \|\psi_{T+}\|^2 d^3\boldsymbol{q} = |\psi_{T-}(\boldsymbol{q}_k = \boldsymbol{q})|^2 *$ Gaussian

GRW's spontaneous collapse

before the "spontaneous collapse":



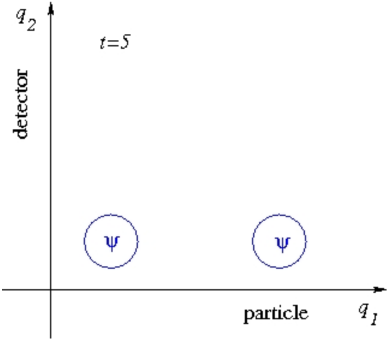
and after:



- In Hilbert space: piecewise deterministic stochastic jump process. ψ_t jumps at random times to random destinations.
- For a single particle, one collapse every 100 million years.
- For 10^4 particles, one collapse every 10,000 years.
- For 10^{23} particles, one collapse every 10^{-7} seconds.
- No-signaling theorem

How GRW theory solves the measurement problem

Evolution of ψ in configuration space of particle + detector:



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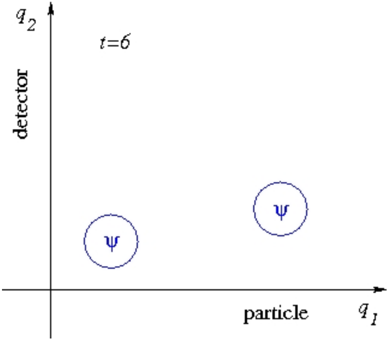

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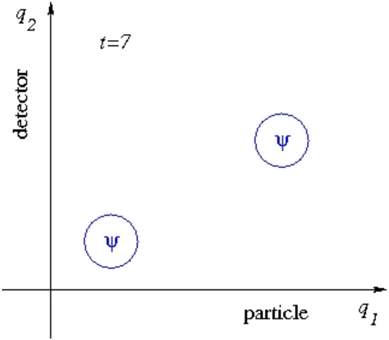

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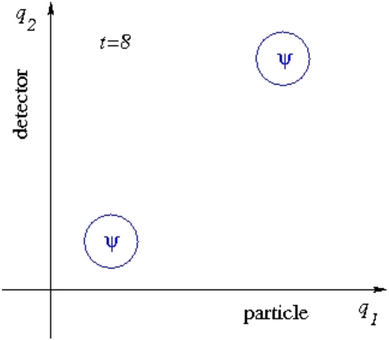

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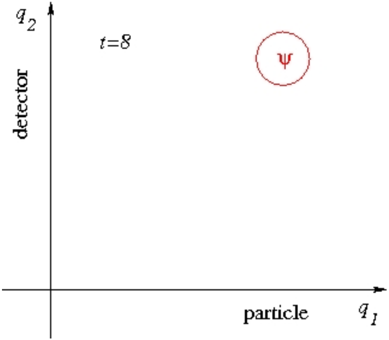
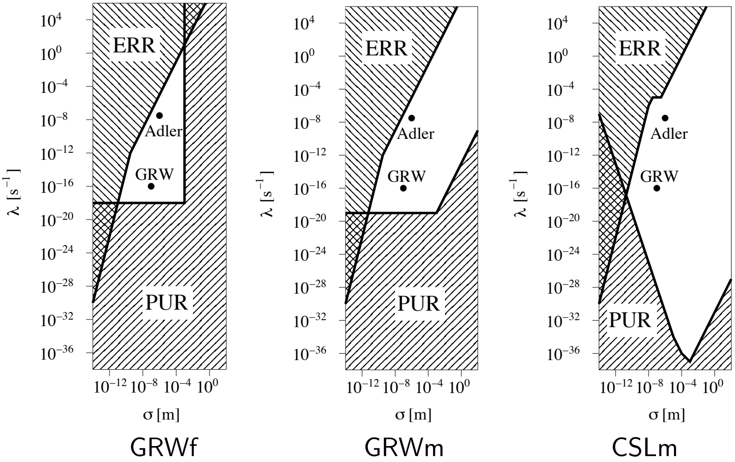


- As soon as a collapse occurs for one particle in the apparatus, the superposition in the test particle is gone as well.
- GRW theory agrees with the standard quantum prediction to an excellent degree of approximation.
- But in principle, the predictions of GRW theory can differ from standard QM.
- For example, in a double slit experiment in which it takes the particle 300 million years to travel from the double slit to the screen, the interference pattern would disappear.
- It is not easy to test GRW against standard QM.
- Dramatic energy increase for much smaller σ values than 10^{-7} m
- Slight energy increase for $\sigma = 10^{-7}$ m

GRW theories are empirically adequate

Their predictions deviate very very slightly from the quantum formalism.



Parameter diagrams (log-log scale). ERR = empirically refuted region, PUR = philosophically unsatisfactory region [Feldmann, Tumulka arXiv:1109.6579]

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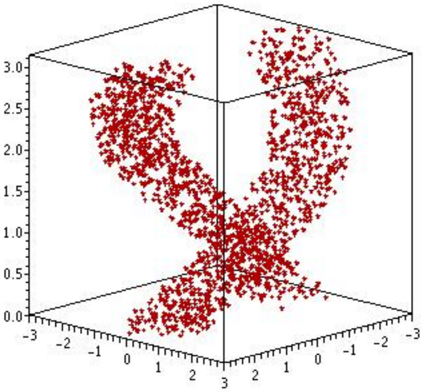
It was long thought that the key to clarity in QM was to avoid talking about ontology and stick to operational statements. That thought has not paid off.

The expression "primitive ontology"

- Def: Primitive ontology is the part of the ontology that represents matter in 3d space (or 4d space-time).
- Example: In Bohmian mechanics, the ontology consists of the particles and the wave function; the primitive ontology consists of the particles.
- I think that for GRW theory to make sense, it needs a primitive ontology.
- The two main proposals are: "flash ontology" and "matter density ontology."

Flash ontology



Instead of particle world lines, there are world points in space-time, called "flashes." A macroscopic object consists of a galaxy of flashes.

If ψ collapses at time T with center $\boldsymbol{q}$ then put a flash at $(T, \boldsymbol{q})$.

Why we need a primitive ontology

- There is a logical gap between saying

$$\text{“}\psi \text{ is the wave function of a live cat”} \tag{1}$$

  and saying

$$\text{“there is a live cat.”} \tag{2}$$

- After all, in Bohmian mechanics, (2) follows from (1) by virtue of a law of the theory, $Q_t \sim |\psi_t|^2$.
- Imagine Bohmian particles guided by a GRW wave function [Allori et al. arXiv:1206.0019]. The particles behave in a catastrophic way, although the wave function looks reasonable. So if you haven't specified the primitive ontology, you don't know what cats or pointers do.

GRW theory in relativistic space-time

Instantaneous collapse

Everybody's first idea:

If collapse is instantaneous (as opposed to propagating at speed c) then it must violate relativity.

That problem is easily avoided [Aharonov and Albert 1981]

For every spacelike hypersurface Σ there is a wave fct $\psi_\Sigma \in \mathscr{H}_\Sigma$.

E.g., $\mathscr{H}_\Sigma = \mathscr{H}_1^{\otimes N}$, $\mathscr{H}_1 = L^2\Big(\Sigma, \mathbb{C}^4, \langle\phi|\psi\rangle = \int_\Sigma d^3x\, \overline{\phi}(x) n_\mu(x) \gamma^\mu \psi(x)\Big)$.

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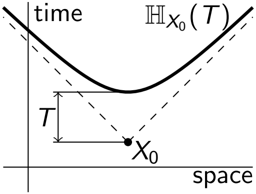
Relativistic GRW model

[Tumulka quant-ph/0406094, quant-ph/0602208, 0711.0035, 2002.00482]

- fixed number N of distinguishable particles
- works also in curved space-time, described here in Minkowski space-time $\mathbb{M} = \mathbb{R}^4$
- works also for interacting particles [2002.00482], described here for non-interacting ones
- works also with matter density ontology [Bedingham et al. 1111.1425], described here with flash ontology
- unitary part of evolution is regarded as given: e.g., free Dirac [arising from $L^2(\mathbb{R}^3, \mathbb{C}^4)$]
- with every spacelike surface Σ there is associated a Hilbert space $\mathscr{H}_\Sigma$
- unitary evolution $U_\Sigma^{\Sigma'}$

The rGRW process for $N = 1$

<u>Given:</u> initial wave fct ψ_0 on some 3-surface Σ_0, seed flash $X_0 \in \mathbb{M}$



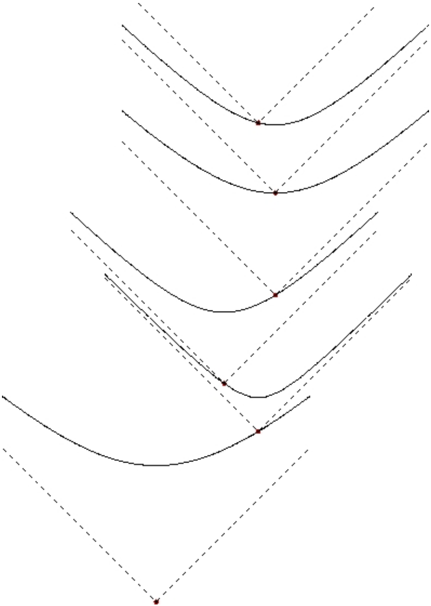
Randomly select next flash $X \in \mathbb{M}$:

- Randomly select waiting time $T \sim \text{Exp}(\lambda)$, T = proper time between X_0 and X, i.e., $X \in \mathbb{H}_{X_0}(T)$
- Evolve $\psi_0 \to \psi_\Sigma$ from Σ_0 to $\Sigma = \mathbb{H}_{X_0}(T)$.
- Randomly select $X \in \Sigma$ with probability density $|\psi_\Sigma|^2 * g$, where $*$ = convolution and g the Gaussian on Σ

$$g(z) = \mathcal{N} \exp\Big(-\frac{\text{dist}_\Sigma(x,z)^2}{2\sigma^2}\Big),$$

$\text{dist}_\Sigma(x,z)$ = spacelike dist. from x to z along Σ, normalization $\int_\Sigma d^3x \, g_x(z) = 1$.

The rGRW process for $N = 1$



Repeat with

ψ_0 replaced by $\dfrac{g_X \psi_\Sigma}{\|g_X \psi_\Sigma\|}$

and X_0 by X.

The rGRW process for $N = 1$

It follows from the definition that the joint distribution of the first n flashes is of the form

$$\mathbb{P}\Big((X_1, \ldots, X_n) \in B\Big) = \langle \psi_0 | G_{1n}(B) | \psi_0 \rangle, \qquad B \subseteq (\mathbb{R}^4)^n$$

where $\psi_0 \in L^2(\Sigma_0)$, and G_{1n} is a positive-operator-valued measure (POVM). Set $G_1 := \lim_{n\to\infty} G_{1n}$.

The rGRW process for $N > 1$

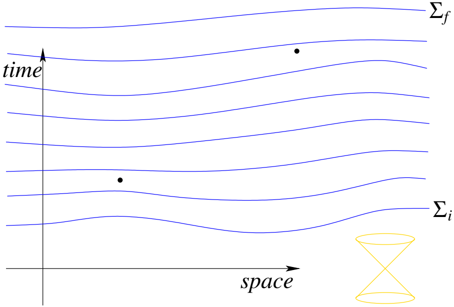
Let the joint probability distribution of the flashes for particles $1 \ldots N$ be

$$\mathbb{P}\Big((X_{11}, X_{12}, \ldots) \in B\Big) = \langle \psi_0 | G_N(B) | \psi_0 \rangle$$

where $\psi_0 \in L^2(\Sigma_0)^{\otimes N}$, and G_N is the product POVM defined by

$$G_N(B_1 \times \cdots \times B_N) = G_1(B_1) \otimes \cdots \otimes G_1(B_N).$$

- We have defined the joint distribution of the flashes.
- random wave function ψ_Σ:
- If the flashes X_{ik} up to Σ are given, ψ_Σ is determined by the initial $\psi_0 \in \mathscr{H}_{\Sigma_0}$: Roughly speaking, collapse ψ at every flash and evolve ψ unitarily in-between.



No signaling

The distribution of the flashes of particle 1 does not depend on the external field A_μ applied to other particles at spacelike separation. It does not depend either on the external field A_μ applied to particle 1 at spacelike separation, except in a neighborhood of size 10^{-7} m and 10^{-8} s.

Nonlocality

The flash process F is nonlocal, i.e., if the space-time regions A and B are spacelike separated then, in general, flashes in A are *not* conditionally independent of those in B, given their common past:

$$\mathbb{P}\Big(F\cap A\Big|F\cap B, F\cap past(A)\cap past(B)\Big) \neq \mathbb{P}\Big(F\cap A\Big|F\cap past(A)\cap past(B)\Big).$$

But there is no fact about who influences whom.

Relativistic GRWf illustrates that a theory can be relativistic and nonlocal.

Thank you for your attention