# Multi-Time Version of the Landau-Peierls Formulation of Quantum Electrodynamics

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Colloquium "Mathematical Physics Regensburg–Munich"
5 February 2021

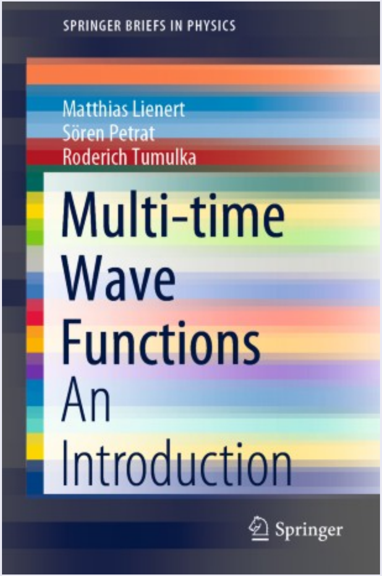
Joint work with Matthias Lienert

These slides are available at https://www.math.uni-tuebingen.de/de/forschung/maphy/personen/roderichtumulka/tumulka-talks

# In memory of Detlef Dürr

"Detlef Dürr's world line began on March 4, 1951,
and ended on January 3, 2021."

# Recent book (November 2020)



An unusual approach to QFT:

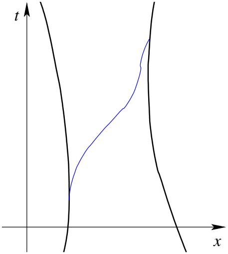
- Let's use wave functions.
- Specifically, a particle-position representation of the quantum state vector.

This will have to do with

- the 1930 model of Landau and Peierls for QED
- multi-time wave functions
- unusual delta functions
- the problem of Born's rule for photons
- interior-boundary conditions.

# My motivation

to explore the possibility of a generalization of Bohmian mechanics with world lines for electrons and photons that can begin and end.



# Landau and Peierls (LP)

**Quantenelektrodynamik im Konfigurationsraum.**

Von **L. Landau** und **R. Peierls** in Zürich.

(Eingegangen am 12. Februar 1930.)

Das elektromagnetische Feld und seine Wechselwirkung mit der Materie wird durch eine Schrödingergleichung im Konfigurationsraum der Lichtquanten beschrieben. Die Resultate sind identisch mit denen von Heisenberg und Pauli.

wrote down a Schrödinger equation for the particle-position representation of the state vector, describing a simplified version of QED, in which

- electrons ($x$) can emit and absorb photons ($y$), $x \leftrightarrows x + y$
- positrons are not considered
- negative energies are not excluded

$$\Phi = \Phi(\boldsymbol{x}_1 ... \boldsymbol{x}_m, \boldsymbol{y}_1 ... \boldsymbol{y}_n) \text{ on } \mathcal{Q} = \bigcup_{m,n=0}^{\infty} \mathcal{Q}^{(m,n)} = \bigcup_{m,n=0}^{\infty} (\mathbb{R}^3)^m \times (\mathbb{R}^3)^n$$

# Free part of LP model

- The free electron equation is the Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0.$$

- The free photon equation is the (complexified) Maxwell equation

$$2\partial^\mu \partial_{[\mu} A_{\nu]} = 0,$$

  where $[\mu\nu]$ means anti-symmetrization in the index pair as in $S_{[\mu\nu]} = \frac{1}{2}(S_{\mu\nu} - S_{\nu\mu})$. That is,

$$\partial^\mu F_{\mu\nu} = 4\pi J_\nu$$

  with field tensor $F_{\mu\nu} = 2\partial_{[\mu} A_{\nu]}$ (so $\partial_{[\lambda} F_{\mu\nu]} = 0$) and source term $J_\nu$.

- E.g., $\Phi^{(1,1)} = \Phi^{(1,1)}_{s,\mu}(\boldsymbol{x}, \boldsymbol{y})$ with $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^3, s \in \{1...4\}, \mu \in \{0...3\}$.
- E.g., disentangled $\Phi^{(1,1)}_{s,\mu}(\boldsymbol{x}, \boldsymbol{y}) = \psi_s(\boldsymbol{x})\, A_\mu(\boldsymbol{y})$.

# Multi-time wave function

- In contrast to the Pauli-Fierz [1938] model, the Landau-Peierls model is relativistic in content, but not written in a Lorentz invariant way: LP used single-time wave fcts.
- single-time $\Phi = \Phi(\boldsymbol{x}_1 \ldots \boldsymbol{x}_n, t)$ with $\boldsymbol{x}_j \in \mathbb{R}^3$
- multi-time $\Psi = \Psi(x_1 \ldots x_n)$ with each $x_j \in \mathbb{R}^4$
  [Eddington 1929, Dirac 1932, Dirac-Fock-Podolsky 1932, Bloch 1934]
- on the set of spacelike space-time configurations

  $$\mathscr{S}_x = \bigcup_{m=0}^{\infty} \Big\{ (x_1 \ldots x_m) \in (\mathbb{M}^4)^m : x_j = x_k \text{ or } x_j \times x_k \text{ for all } j, k \Big\}$$

  $\times$ means spacelike separated
- For us, $\Psi$ on $\mathscr{S}_{xy}$,

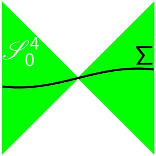
$$\Psi^{(m,n)}_{s_1 \ldots s_m, \mu_1 \ldots \mu_n}(x_1 \ldots x_m, y_1 \ldots y_n) \, .$$

## Multi-time LP equations

# 3d Dirac delta distribution

- ordinary $\delta^3(\boldsymbol{x}) = \delta(x_1)\,\delta(x_2)\,\delta(x_3)$, $\quad \int_{\mathbb{R}^3} d^3\boldsymbol{x}\,\delta^3(\boldsymbol{x})\,f(\boldsymbol{x}) = f(\boldsymbol{0})$
- introduce $D = \delta^3_\mu$ on $\mathscr{S}_0^4 = \{x \in \mathbb{M}^4 : x = 0 \text{ or } x \times 0\}$ that can be integrated over any (smooth) Cauchy surface $0 \in \Sigma \subset \mathbb{M}^4$ against (smooth) $f : \Sigma \to \mathbb{C}$,

$$\int_\Sigma D\,f = \int_\Sigma V(d^3x)\,n_\rho(x)\,D^\rho(x)\,f(x) = f(0) \quad (*)$$




- Proof that such a $D$ exists: On $\mathbb{M}^4$ define $\tilde{D}^\rho(x^0,\boldsymbol{x}) = \big(\delta^3(\boldsymbol{x}),0,0,0\big)$ and set $D = \tilde{D}\big|_{\mathscr{S}_0^4}$. A calculation confirms (*). Likewise for any Lorentz transform of $\tilde{D}$.
- Alternative proof: Consider the free Dirac propagator $S$ on $\mathbb{M}^4$,

$$\psi(x) = \int_{\mathbb{R}^3} d^3\boldsymbol{y}\,S(x-(0,\boldsymbol{y}))\,\gamma^0\,\psi_0(\boldsymbol{y})\,.$$

$S$ is Lorentz inv., so $\int_\Sigma S\gamma^\mu f = \int_{\Sigma'} S\gamma^\mu f' = I_4 f(0)$. Thus, $S\gamma^\mu = I_4\,D^\mu$.

# Multi-time LP equations [Lienert, Tumulka 2020]

$$(i\gamma_j^\mu \partial_{x_j,\mu} - m_x)\Psi^{(m,n)}(x_1 ... x_m, y_1 ... y_n) = e\sqrt{n+1}\,\gamma_j^\rho\,\Psi^{(m,n+1)}_{\mu_{n+1}=\rho}(x_1 ... x_m, y_1 ... y_n, x_j) \quad (1)$$

$$2\partial^\mu_{y_k}\partial_{y_k,[\mu}\Psi^{(m,n)}_{\mu_k=\nu]}(x_1 ... x_m, y_1 ... y_n) = \frac{4\pi e}{\sqrt{n}}\sum_{j=1}^m \delta^3_\mu(y_k - x_j)\,\gamma_j^\mu\gamma_{j\nu}\,\Psi^{(m,n-1)}_{\widehat{\mu_k}}(x_1 ... x_m, y_1 ... y_{k-1}, y_{k+1} ... y_n) \quad (2)$$

$$\sum_{\mu_k=1}^3 \partial^{\mu_k}_{y_k}\Psi^{(m,n)}_{\mu_k}(x_1 ... x_m, y_1 ... y_n) = 0 \quad (3)$$

# Multi-time LP equation (1)

$$(i\gamma_j^\mu \partial_{x_j,\mu} - m_x)\Psi^{(m,n)}(x_1...x_m, y_1...y_n) = e\sqrt{n+1}\,\gamma_j^\rho\,\Psi^{(m,n+1)}_{\mu_{n+1}=\rho}(x_1...x_m, y_1...y_n, x_j) \quad (1)$$

Compare to the Dirac equation in an external electromagnetic field with vector potential $A_\mu(x)$,

$$(i\gamma^\mu\partial_\mu - m_x)\psi(x) = e\,\gamma^\rho A_\rho(x)\,\psi(x)\,.$$

For multi-time wave fct of non-interacting particles, that would give

$$(i\gamma_j^\mu\partial_{x_j,\mu} - m_x)\Psi^{(m,n)}(x_1...x_m, y_1...y_n) = e\,\gamma_j^\rho A_\rho(x_j)\Psi^{(m,n)}(x_1...x_m, y_1...y_n)\,.$$

The factor $\sqrt{n+1}$ in (1) is due to our normalization convention (use of ordered configurations). **The role of the vector potential $A_\mu$ is played by the wave fct of the next photon.**

# Multi-time LP equation (2)

$$2\partial^{\mu}_{y_k}\partial_{y_k,[\mu}\Psi^{(m,n)}_{\mu_k=\nu]}(x_1...x_m,y_1...y_n) = \frac{4\pi e}{\sqrt{n}}\sum_{j=1}^{m}\delta^3_\mu(y_k-x_j)\,\gamma^\mu_j\gamma_{j\nu}\,\Psi^{(m,n-1)}_{\widehat{\mu_k}}(x_1...x_m,y_1...y_{k-1},y_{k+1}...y_n) \quad (2)$$

Compare to the Maxwell eq

$$2\partial^\mu\partial_{[\mu}A_{\nu]}(y) = 4\pi J_\nu(y).$$

For multi-time wave fct of non-interacting particles and "$A_\mu = \Psi_\mu$,"

$$2\partial^{\mu}_{y_k}\partial_{y_k,[\mu}\Psi^{(m,n)}_{\mu_k=\nu]}(x_1...x_m,y_1...y_n) = 4\pi J_{k,\nu}.$$

In (2),

$$J_{k,\nu} = \frac{e}{\sqrt{n}}\sum_{j=1}^{m}\delta^3_\mu(y_k-x_j)\,\gamma^\mu_j\gamma_{j\nu}\,\Psi^{(m,n-1)}_{\widehat{\mu_k}}(x_1...x_m,y_1...y_{k-1},y_{k+1}...y_n).$$

# Multi-time LP equation (3)

$$\sum_{\mu_k=1}^{3} \partial_{y_k}^{\mu_k} \Psi_{\mu_k}^{(m,n)}(x_1 ... x_m, y_1 ... y_n) = 0 \tag{3}$$

Compare to the Coulomb gauge condition

$$\sum_{\mu=1}^{3} \partial^{\mu} A_{\mu} = 0.$$

in view of "$A_\mu = \Psi_\mu$."

# Field operators $\hat{\psi}$, $\hat{A}_\mu$

$$\Psi^{(m,n)}_{s_1\ldots s_m,\mu_1\ldots\mu_n}(x_1\ldots x_m, y_1\ldots y_n) = \frac{1}{\sqrt{m!n!}}\left\langle \emptyset \middle| \hat{\psi}_{s_1}(x_1)\cdots\hat{\psi}_{s_m}(x_m)\hat{A}_{\mu_1}(y_1)\cdots\hat{A}_{\mu_n}(y_n) \middle| \varphi \right\rangle$$

## Proposition [Lienert, Tumulka 2020]

(1), (2), (3) on $\mathscr{S}_{xy}$ follow from $\hat{\psi}_s(x)|\emptyset\rangle = 0$,

$$\sum_{s'}\Big(i\gamma^{\mu s'}_{s}\left[\partial_\mu + ie\hat{A}_\mu(x)\right] - m_x\Big)\hat{\psi}_{s'}(x) = 0$$

$$2\partial^\mu\partial_{[\mu}\hat{A}_{\nu]}(x) = 4\pi e\,\overline{\hat{\psi}(x)}\,\gamma_\nu\,\hat{\psi}(x)$$

$$\sum_{\mu=1}^{3}\partial^\mu\hat{A}_\mu(x) = 0$$

and canonical comm. relations $\left\{\overline{\hat{\psi}(x)}^s, \hat{\psi}_{s'}(x')\right\} = \gamma^{\mu s}_{s'}\,\delta^3_\mu(x-x')$ etc.

The multi-time approach allows for dealing with gauge more clearly.

- A particular choice of gauge was built into LP's formula for the Hamiltonian.
- In the multi-time approach, the gauge condition (3) is a separate equation from (1) and (2).
- What is more, in the multi-time approach, we can switch between the $A_\mu$-representation and the $F_{\mu\nu}$-representation:

Let $d^\rho_{\mu\nu} = \delta^\rho_\nu \partial_\mu - \delta^\rho_\mu \partial_\nu$. Then $F_{\mu\nu} = d^\rho_{\mu\nu} A_\rho$, and $d = d^\rho_{\mu\nu}$ can be applied to a single $y_k$.
For example, consider a quantum state of 1 electron and 1 photon that are not entangled, and drop interaction (creation/annihilation) terms:

$$\Psi^{(1,1)}_{s\mu}(x,y) = \psi_s(x)\, A_\mu(y).$$

Then $d_y \Psi^{(1,1)}(x,y) = \psi_s(x)\, F_{\mu\nu}(y)$. Nothing like that is possible in the single-time approach, as any time derivative would also affect $x$ (and $\psi$).

More on gauge later.

# Agreement with LP

- LP assumed for every photon only positive frequencies.
- LP considered the $F_{\mu\nu}$-representation instead of the $A_\mu$-representation and called it $F$. In the multi-time framework,

$$F^{(m,n)}_{s_1\ldots s_m,\mu_1\nu_1\ldots\mu_n\nu_n}(x_1\ldots x_m, y_1\ldots y_n) := d_{y_1}\cdots d_{y_n}\Psi^{(m,n)}(x_1\ldots x_m, y_1\ldots y_n)\,.$$

- Using Coulomb gauge, positive frequency and square-integrability, one can reconstruct $A_\mu = (A_0, \boldsymbol{A})$ from $F_{\mu\nu}$ via pseudo-differential operators

$$A_0 = -\mathrm{div}\,\Delta^{-1}\boldsymbol{E}, \qquad \boldsymbol{A} = -\mathrm{curl}\,\Delta^{-1}\boldsymbol{B}.$$

- Use this for closing (1) for $F$.
- Restrict to equal times, $t = x_1^0 = x_2^0 = \ldots = x_m^0 = y_1^0 = \ldots = y_n^0$.
- Then one obtains from (1), (2), (3) the LP (1-time) Hamiltonian and the LP constraint up to
  - factors of $\sqrt{n}$ that they seem to have forgotten
  - a factor of 2 in one place that I think is mistaken.

# Interior-boundary conditions

# Interior-boundary conditions (IBCs) (i)

Another advantage of the particle-position representation:

In non-relativistic models of $x \leftrightarrows x + y$, the $\delta^3$ in the creation term makes the Hamiltonian UV divergent. The divergence disappears if we demand a certain condition on the wave function $\Phi^{(m,n)}(\boldsymbol{x}_1 \ldots \boldsymbol{x}_m, \boldsymbol{y}_1 \ldots \boldsymbol{y}_n)$ at configurations with $\boldsymbol{x}_j = \boldsymbol{y}_k$, the IBC.

[Teufel, Tumulka 2015; Lampart, Schmidt, Teufel, Tumulka 2017; Lampart 2018]

In the simplified case of a single $x$-particle fixed at $\mathbf{0} \in \mathbb{R}^3$, the IBC reads, for all unit vectors $\boldsymbol{\omega} \in \mathbb{S}^2$,

$$\lim_{r \searrow 0} \left( r \Phi^{(n+1)}(\boldsymbol{y}_1 \ldots \boldsymbol{y}_n, r\boldsymbol{\omega}) \right) = -\frac{e\, m_y}{2\pi\sqrt{n+1}} \, \Phi^{(n)}(\boldsymbol{y}_1 \ldots \boldsymbol{y}_n) \, .$$

[Moshinsky 1951; Teufel, Tumulka 2015]

# Interior-boundary conditions (IBCs) (ii)

**Known:** the 8 Maxwell eqs are 6 time evolution eqs and 2 constraints, $\operatorname{div}\boldsymbol{B} = 0$ and $\operatorname{div}\boldsymbol{E} = 4\pi J_0$. For $J_0(\boldsymbol{y}) = e\sum_{j=1}^{m}\delta^3(\boldsymbol{y}-\boldsymbol{x}_j)$, we obtain for small $r > 0$ that

$$4\pi e = \int_{B_r(\boldsymbol{x}_j)} d^3\boldsymbol{y}\,\operatorname{div}\boldsymbol{E}(\boldsymbol{y}) = r^2\int_{\mathbb{S}^2} d^2\boldsymbol{\omega}\,\boldsymbol{\omega}\cdot\boldsymbol{E}(\boldsymbol{x}_j + r\boldsymbol{\omega})$$

by the Ostrogradski-Gauss integral theorem. Now let $r \to 0$.

**The same consideration applied to** (2) leads to an IBC:

$$\lim_{r\to 0}\int_{\mathbb{S}^2} d^2\boldsymbol{\omega}\, r^2 \sum_{i=1}^{3}\omega^i \partial_{y_{n+1},[i}\Psi^{(m,n+1)}_{\mu_{n+1}=0]}(x_1 \ldots x_m, y_1 \ldots y_n, (x_j^0, \boldsymbol{x}_j + r\boldsymbol{\omega})) = \frac{4\pi e}{\sqrt{n}}\,\Psi^{(m,n)}(x_1 \ldots x_m, y_1 \ldots y_n).$$

Whether that helps with any UV problem remains to be seen.

# Questions of gauge

# Status of choice of gauge (i)

- LP thought the gauge condition was a matter of “convenience.” This sounds as if any gauge condition could be easily replaced with any other gauge condition, like a choice of basis or of coordinates.
- Likewise, we initially expected that any we could drop all gauge conditions. We expected we could drop (3), and that the solution would be unique up to a gauge transformation. Alas, how wrong!

# Status of choice of gauge (ii)

Consider, e.g., at $t = 0$ a disentangled state with 1 electron, 1 photon,

$$\Psi^{(1,1)}_{s\mu}(0, \boldsymbol{x}, 0, \boldsymbol{y}) = \psi_s(\boldsymbol{x})\, A_\mu(\boldsymbol{y})$$
$$\Psi^{(1,0)}_s(0, \boldsymbol{x}) = 0.$$

Then, by (1),

$$\partial_t \Psi^{(1,0)}(\boldsymbol{x}) = e\gamma^\mu \psi(\boldsymbol{x}) A_\mu(\boldsymbol{x}).$$

So, it seems the prob. of 1 electron at $\boldsymbol{x}$ and 0 photons at time $dt$ is

$$\rho(\boldsymbol{x}) = \left|\Psi^{(1,0)}(dt, \boldsymbol{x})\right|^2 = e^2 \left|\gamma^\mu \psi(\boldsymbol{x}) A_\mu(\boldsymbol{x})\right|^2.$$

Now replace $A_\mu$ by another $\tilde{A}_\mu$ that is gauge equivalent. Then (observable) probabilities have changed.

**Upshot**

(3) is not like a choice of basis or coordinates, it is one of the defining eqs of the theory. Among possible equations for (3), only one is correct.

# The problem of Born's rule for photons

# The problem of Born's rule for photons

- Born's rule: Specifies prob density $\rho$ or current $j^\mu$ in terms of wave fct.
- Let's focus on single particle.
- For Dirac wave fct $\psi : \mathbb{R}^4 \to \mathbb{C}^4$, $\quad j_{\mathrm{D}}^\mu(x) = \overline{\psi}(x)\,\gamma^\mu\,\psi(x)$.
- For photon, $j_{\mathrm{ph}}^\mu = ?$
- There is a convincing answer for plane waves:

# $j^\mu_{\rm ph}$ for plane waves

- Suppose $A_\mu(x) = a_\mu\, e^{ik_\lambda x^\lambda}$, $\quad F_{\mu\nu}(x) = 2i\, a_{[\mu}\, k_{\nu]}\, e^{ik_\lambda x^\lambda}$ with future-lightlike $k^\mu$ and $k^\mu a_\mu = 0 \quad \Leftarrow \quad \partial^\mu F_{\mu\nu} = 0$.
- In the classical regime (many photons with equal wave fcts), photon wave fct becomes electromagnetic field.
- Classically, energy density is $T_{\mu\nu} = \mathrm{Re}\big[F^*_{\mu\lambda} F_\nu{}^\lambda\big] - \frac{1}{4} g_{\mu\nu} F^*_{\lambda\rho} F^{\lambda\rho}$.
- Thus, here, $T_{\mu\nu} = a^*_\lambda\, a^\lambda\, k_\mu k_\nu$.
- If each photon has momentum $\hbar k^\mu$, then $T_{\mu\nu} = \hbar k_\mu j_\nu$ up to constant factor.
- Thus,

$$j^\mu = a^*_\lambda a^\lambda\, k^\mu \tag{4}$$

  up to constant factor.
- Also OK for local plane waves (as occur in the scattering regime).
- But not every Maxwell field is a local plane wave. So the problem remains.

# The desired law for $j^\mu_{\text{ph}}$ should have these properties:

1. The expression is quadratic in $A_\mu$ and its derivatives.
2. The expression is local, i.e., $j^\nu(x)$ depends only on $A_\mu$ and its derivatives at $x$.
3. $j^\mu$ is future timelike-or-lightlike.
4. $\partial_\mu j^\mu = 0$ if $A_\mu$ obeys the free Maxwell equations.
5. For a plane wave, $j^\mu$ agrees with (4) up to a constant factor.
6. No choices need to be made, i.e., if the law requires a special gauge or Lorentz frame then it also specifies this gauge or Lorentz frame.
7. The law can be generalized to curved space-time.

Several proposals for $j^\mu$ have been made [Landau, Peierls 1930; Bialynicki-Birula 1994; Kiessling, Tahvildar-Zadeh 2018; etc.],
none of them satisfies all of the properties above;
some of them may be useful approximations.
It seems the correct answer has not been found yet.

Hope/guess: Maybe there is a formula for $j^\nu$ in terms of $A_\mu$ and its derivatives that applies only in a particular gauge.

# Consistency

Multi-time equations can be inconsistent.

**Theorem** [Petrat, Tumulka 2013]

For $\Psi(q, t_1 \ldots t_N)$ with fixed number $N$ of time variables, the multi-time system

$$i\frac{\partial \Psi}{\partial t_k} = H_k \Psi$$

is consistent if and only if

$$\Big[ i\partial_{t_j} - H_j, i\partial_{t_k} - H_k \Big] = 0.$$

# Consistency results for multi-time QFT models (i)

A model of QED due to Dirac, Fock, and Podolsky [1932] involves 1 time variable for each of $M$ Dirac particles and 1 time variable for the quantized electromagnetic field.

**Theorem**

The equations are formally consistent [Bloch 1934]. After introducing a UV cut-off, also rigorously consistent [Nickel, Deckert 2019].

# Consistency results for multi-time QFT models (ii)

Another model [Petrat, Tumulka 2013] involves $\Psi^{(m,n)}(x_1 \ldots x_m, y_1 \ldots y_n)$ with $m+n$ time variables, $x \leftrightarrows x + y$, both $x$ and $y$ Dirac particles: On $\mathscr{S}_{xy}$,

$$(i\gamma_j^\mu \partial_{x_j,\mu} - m_x)\Psi^{(m,n)}(x_1 \ldots x_m, y_1 \ldots y_n) = e\sqrt{n+1}\, g^{r_{n+1}}\, \Psi^{(m,n+1)}_{r_{n+1}}(x_1 \ldots x_m, y_1 \ldots y_n, x_j) \qquad (1')$$

$$(i\gamma_k^\mu \partial_{y_k,\mu} - m_y)\Psi^{(m,n)}(x_1 \ldots x_m, y_1 \ldots y_n) = \frac{e}{\sqrt{n}} \sum_{j=1}^{m} \delta^3_\mu(y_k - x_j)\, h^\mu_{r_k}\, \Psi^{(m,n-1)}_{\widehat{r_k}}(x_1 \ldots x_m, y_1 \ldots y_{k-1}, y_{k+1} \ldots y_n) \qquad (2')$$

with suitable coefficients $g, h$.

**Theorem**

The equations are formally consistent [Petrat, Tumulka 2013]. After introducing a UV cut-off, also rigorously consistent [Lill, Nickel, Tum. 2020].

The proof makes use of propagation locality and interaction locality.

### Propagation locality

Wave fcts cannot propagate faster than light.

### Interaction locality

No interaction terms in the time evolution law for the wave fct between spacelike separated regions.

### Conjecture [Lienert, Tumulka 2020]

For every 1-time Hamiltonian $H$ satisfying propagation locality and interaction locality, there is a unique consistent multi-time evolution on $\mathscr{S}$ satisfying propagation locality and interaction locality and agreeing with the $H$-evolution on horizontal (i.e., simultaneous) configurations.

Interaction locality is OK for LP. However, while the Maxwell eqs are propagation local for $F_{\mu\nu}$, it may depend on the gauge whether the evolution of $A_\mu$ is. For the time being, the problem remains open. (Connected to the problem of Born's rule: One usually proves propagation locality by using that $j^\mu$ is timelike-or-lightlike.)

# Outlook

# Outlook

- LP's approach appears as simple and natural
- Becomes even more natural in the multi-time framework
- Open problems:
  - Lorentz invariant formulation of the IBC
  - Which equation for (3)?
  - Born rule for photons
  - Consistency proof
  - What about positrons, Dirac sea?

## Thank you for your attention