

The Bohmian Approach to the Problems of Cosmological Quantum Fluctuations

Roderich Tumulka



La Sapienza, Rome, 3 July 2018

15th Marcel Grossmann Meeting

Joint work with Sheldon Goldstein and Ward Struyve,
supported by the John Templeton Foundation

Two problems of quantum fluctuations in cosmology

- Late universe: Want to avoid Boltzmann brains that arise by fluctuation
- Early universe: Want seeds of galaxy formation from quantum superposition

Bohmian mechanics helps with both problems.

Plan of the talk

- The problem of Boltzmann brains
- What is Bohmian mechanics?
- How Bohmian mechanics helps against Boltzmann brains

The problem of Boltzmann brains

Boltzmann brains

[Albrecht 2002; Dyson, Kleban, and Susskind 2002; Albrecht and Sorbo 2004; Bousso and Freivogel 2007; Boddy, Carroll, and Pollack 2014, 2015]

- A “Boltzmann brain” is this: Let M be the present macro-state of your brain. For a classical gas in thermal equilibrium, it has probability 1 that after sufficient waiting time, some atoms will “by coincidence” (or “by fluctuation”) come together in such a way as to form a subsystem in a micro-state belonging to M . That is, this brain comes into existence not by childhood and evolution of life forms, but by coincidence; this brain has memories (duplicates of your present memories), but they are false memories: the events described in the memories never happened to this brain!
- Boltzmann brains are, of course, very unlikely. But they will happen if the waiting time is long enough, and they will happen more frequently if the system is larger (bigger volume, higher number of particles).

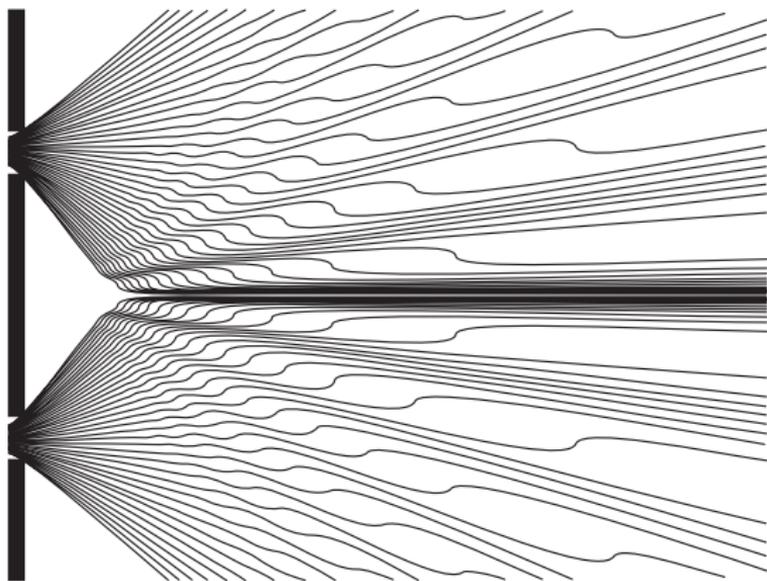
The problem of Boltzmann brains

- The problem is this: If the universe continues to exist forever, and if it reaches universal thermal equilibrium at some point, then the overwhelming majority of brains in the universe will be Boltzmann brains. According to the “Copernican principle,” we should see what a typical observer sees. Thus, the theory predicts that we are Boltzmann brains. But we are not. (Because most Boltzmann brains find themselves surrounded by thermal equilibrium, not by other intelligent beings on a planet.)
- How can any of our serious theories avoid making this incorrect prediction?

Concrete version of the Boltzmann brain problem

- It is expected (e.g., from Λ CDM) that the late universe will be close to [de Sitter space-time](#), and the state of matter will be close (in terms of local observables) to the [Bunch-Davies vacuum](#), a quantum state invariant under the isometries of de Sitter space-time. The probability distribution it defines on configuration space gives $> 99\%$ weight to thermal equilibrium configurations, but positive probability to brain configurations, in fact $> 99\%$ probability to configurations containing brains if 3-space is large enough (in particular if infinite).
- **Question:** Does this mean there are Boltzmann brains in the Bunch-Davies vacuum? [What is the significance of this particular wave function for reality?](#) Does a stationary state mean that nothing happens?
- Or does the factual situation visit different configurations over time according to $|\psi|^2$?

Bohmian mechanics



Picture: Gernot Bauer (after Chris Dewdney)

wave-particle duality (in the literal sense)

For non-relativistic QM: [Slater 1923, de Broglie 1926, Bohm 1952, Bell 1966]

- Dynamical laws:

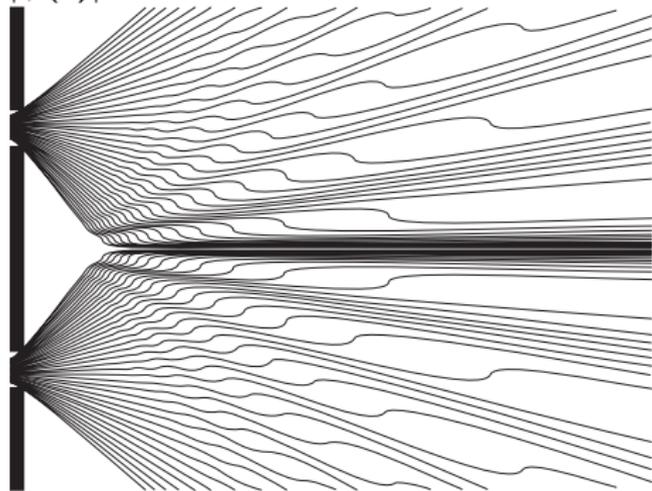
$$\frac{d\mathbf{Q}_k}{dt} = \frac{\hbar}{m_k} \nabla_k \text{Im} \log \psi(\mathbf{Q}_1(t), \dots, \mathbf{Q}_N(t)) \quad (1)$$

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \quad (2)$$

- The law of motion (1) is equivalent to $dQ/dt = j/\rho$, where $Q = (\mathbf{Q}_1, \dots, \mathbf{Q}_N)$ is the configuration, $\rho = |\psi|^2$ is the standard probability density, and j is the standard probability current vector field in configuration space.
- Quantum equilibrium assumption:

$$Q(t=0) \text{ is random with distribution density } |\psi(t=0)|^2. \quad (3)$$

Equivariance theorem: It follows that at any time, $Q(t)$ has distribution $|\psi(t)|^2$.



Central fact

Inhabitants of a Bohmian world would observe outcomes in agreement with the predictions of quantum mechanics.

How Bohmian mechanics helps with Boltzmann brains

Freezing in Bohmian mechanics

- In Bohmian mechanics, there is the phenomenon of “freezing.”

Theorem

If ψ is a non-degenerate eigenstate of a real H then the Bohmian configuration does not move.

- That is because the conjugate of ψ must be another eigenstate with the same eigenvalue, so ψ must be real up to a global phase. As a consequence, Bohmian velocities (and particle creation rates) vanish.
- (Surprising because the momentum distribution is not concentrated on the origin. In Bohmian mechanics, momentum corresponds not to the instantaneous velocity but to the asymptotic velocity that the particle would reach if the potential were turned off.)
- It follows that, if non-relativistic Bohmian mechanics were true, and if the late universe were in a non-degenerate eigenstate, then the configuration would be frozen. Arguably, the Boltzmann brain problem is absent then, as the brain, even if it existed, would not be functioning.

Concrete Model

- De Sitter space-time has metric

$$ds^2 = dt^2 - e^{2Ht} \delta_{ij} dx^i dx^j$$

(H = Hubble parameter = expansion speed; $i, j = 1 \dots 3$)

- Simple quantum field theory: Hermitian scalar quantum field $\varphi(\mathbf{x}, t)$
- Wave functional $\Psi(\varphi, t)$ on space of field configurations φ
- common rescaling: $dt = e^{Ht} d\eta$ (η = conformal time), $y = e^{Ht} \varphi$
- $-\infty < t < \infty$ but $-\infty < \eta < 0$: $t \rightarrow \infty$ corresponds to $\eta \rightarrow 0^-$
- Schrödinger equation:

$$i \frac{\partial \Psi}{\partial \eta} = \frac{1}{2} \int d^3x \left[-\frac{\delta^2}{\delta y(\mathbf{x})^2} + \delta_{ij} \partial^i y(\mathbf{x}) \partial^j y(\mathbf{x}) + \frac{i}{\eta} \left(\frac{\delta}{\delta y(\mathbf{x})} y(\mathbf{x}) + y(\mathbf{x}) \frac{\delta}{\delta y(\mathbf{x})} \right) \right] \Psi$$

Concrete Model

[Hiley and Aziz Mufti 1995; Pinto-Neto, Santos, and Struyve 2012]

- Bohmian model with field ontology: actual field configuration $\varphi(\mathbf{x}, t)$

$$\frac{dy(\mathbf{x})}{d\eta} = \frac{\delta \text{Im} \log \Psi}{\delta y(\mathbf{x})} - \frac{1}{\eta} y(\mathbf{x})$$

- In terms of Fourier modes $y_{\mathbf{k}}$, (\mathbb{R}^{3+} = half space, note $y_{-\mathbf{k}} = y_{\mathbf{k}}^*$)

$$i \frac{\partial \Psi}{\partial \eta} = \int_{\mathbb{R}^{3+}} d^3 k \left[-\frac{\delta^2}{\delta y_{\mathbf{k}}^* \delta y_{\mathbf{k}}} + k^2 y_{\mathbf{k}}^* y_{\mathbf{k}} + \frac{i}{\eta} \left(\frac{\delta}{\delta y_{\mathbf{k}}^*} y_{\mathbf{k}}^* + y_{\mathbf{k}} \frac{\delta}{\delta y_{\mathbf{k}}} \right) \right] \Psi$$

$$\frac{dy_{\mathbf{k}}}{d\eta} = \frac{\delta \text{Im} \log \Psi}{\delta y_{\mathbf{k}}^*} - \frac{1}{\eta} y_{\mathbf{k}}$$

Freezing in the Bunch-Davies state

[Goldstein, Struyve, and Tumulka 2015]

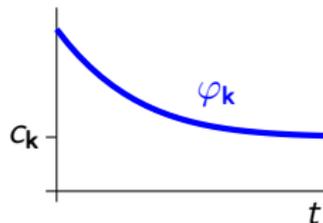
- Bunch-Davies state: ($f = f_k(\eta) = \sqrt{1 + 1/k^2\eta^2}/\sqrt{2k}$)

$$\Psi = \prod_{\mathbf{k} \in \mathbb{R}^{3+}} \frac{1}{\sqrt{2\pi f}} \exp \left\{ -\frac{1}{2f^2} |y_{\mathbf{k}}|^2 + i \left[\left(\frac{f'}{f} + \frac{1}{\eta} \right) |y_{\mathbf{k}}|^2 - \text{phase}(k, \eta) \right] \right\}$$

- Solution to Bohmian eq. of motion:

$$y_{\mathbf{k}}(\eta) = \tilde{c}_{\mathbf{k}} f_k(\eta) \text{ or}$$

$$\varphi_{\mathbf{k}}(t) = c_{\mathbf{k}} \sqrt{1 + k^2 \exp(-2Ht)/H^2} \quad (4)$$



- Note that $\exists \lim_{t \rightarrow \infty} \varphi_{\mathbf{k}}(t) = c_{\mathbf{k}}$ (freezing).
- In fact, at any time only the modes with wave lengths large compared to the Hubble distance $1/H$ are frozen. But the simple behavior (4) is as good as freezing for removing the Boltzmann brain problem: Too simple to support the complex behavior of a functioning brain.

Freezing in a generic state

- Ψ will not be close to the Bunch-Davies state in Hilbert space. It will look locally similar, but Bohmian mechanics depends nonlocally on the wave function.
- This issue is taken care of by the following

Theorem

[Ryssens 2012; Tumulka 2015]

For a large class of wave functions and most initial field configurations, the asymptotic long-time behavior of φ is

$$\varphi_{\mathbf{k}}(t) \approx c_{\mathbf{k}} \sqrt{1 + k^2 \exp(-2Ht)/H^2} \quad \text{for } t > t_0,$$

where t_0 is independent of \mathbf{k} (but depends on the wave function). In particular, $\exists \lim_{t \rightarrow \infty} \varphi_{\mathbf{k}}(t)$.

Idea of proof

- consider a single mode \mathbf{k}
- rescale field variable $z = \gamma(\eta)^{-1}y$
- rescale and phase-transform wave function,

$$\Phi(z, \eta) = e^{\alpha(\eta) + i\beta(\eta)z^*z} \Psi(\gamma(\eta)z, \eta)$$

- rescale time $d\tau = \gamma^{-2}d\eta$
- If scaling functions α, β, γ are chosen suitably, the evolution of Φ reduces to a non-relativistic Schrödinger equation in a 2d harmonic oscillator potential

$$i \frac{\partial \Phi}{\partial \tau} = - \frac{\partial^2 \Phi}{\partial z^* \partial z} + \omega^2 z^* z \Phi$$

and non-relativistic Bohmian equation of motion

$$\frac{dz}{d\tau} = \frac{\partial \text{Im} \log \Phi(z, \tau)}{\partial z^*}$$

which do not become singular as $\tau \rightarrow 0^-$ ($\Leftrightarrow \eta \rightarrow 0^- \Leftrightarrow t \rightarrow \infty$).

- Thus, $\lim_{\tau \rightarrow 0^-} z(\tau)$ exists, so, for τ sufficiently close to 0,

$$z(\tau) \approx \text{const.} \Leftrightarrow y(\eta) \approx \gamma(\eta) \times \text{const.} \Leftrightarrow \varphi(t) \approx \sqrt{1 + \exp(-2Ht)}.$$

- For almost any ψ , freezing occurs: the Bohmian configuration does not move, or moves in a simple way. While there is a positive probability for a brain configuration to occur, this subsystem would not function as a brain because it is frozen. (In fact, the probability of a brain configuration in our Hubble volume is tiny.)
- Thus, **Boltzmann brains do not occur in Bohmian mechanics**, at least according to this particular model.
- By the way, for Everett's [1957] many-worlds interpretation of QM, Boddy, Carroll, and Polack [2014, 2015] have argued that in the stationary Bunch-Davies state nothing moves, but Wallace [2014] disagrees.

Thank you for your attention

- W. Ryssens: *On the Quantum-to-Classical Transition of Primordial Perturbations*. Master thesis, Department of Physics and Astronomy, Katholieke Universiteit Leuven (2012)
- R. Tumulka: Long-Time Asymptotics of a Bohmian Scalar Quantum Field in de Sitter Space-Time.
General Relativity and Gravitation **48**: 2 (2016)
<http://arxiv.org/abs/1507.08542>
- S. Goldstein, W. Struyve, and R. Tumulka: The Bohmian Approach to the Problems of Cosmological Quantum Fluctuations. To appear in A. Ijjas and B. Loewer (editors), *Guide to the Philosophy of Cosmology*, Oxford University Press (2019)
<http://arxiv.org/abs/1508.01017>