On the Measurability of Arrival Times

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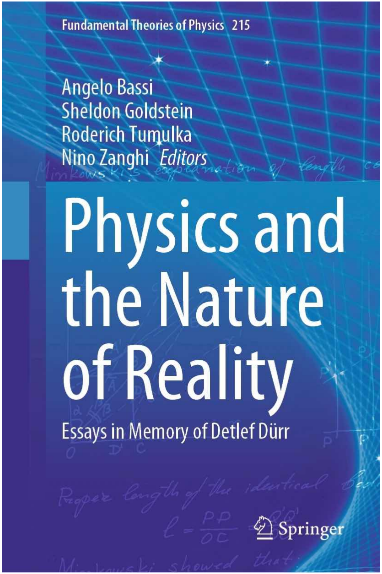
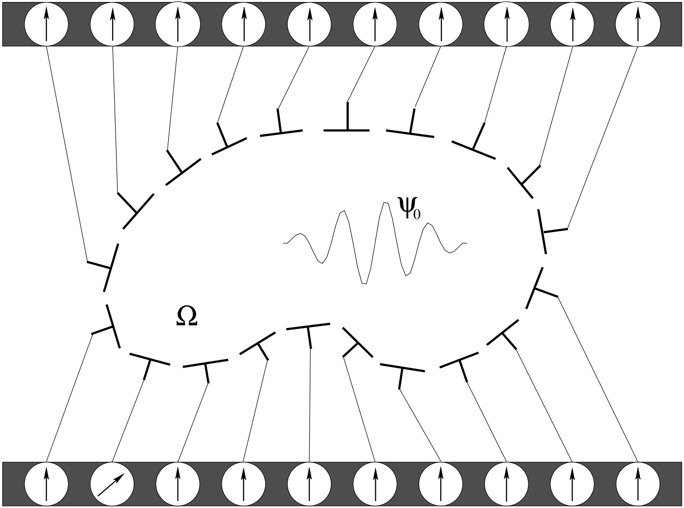
Seminar at Rutgers University, 4 April 2024

Joint work with Shelly Goldstein and Nino Zanghì
http://arxiv.org/abs/2309.11835
another paper coming up this month

Recent book


Fundamental Theories of Physics 215
Angelo Bassi
Sheldon Goldstein
Roderich Tumulka
Nino Zanghì Editors
Physics and the Nature of Reality
Essays in Memory of Detlef Dürr
Springer

Problem of detection time and place



$T \in [0, \infty), \boldsymbol{X} \in \partial\Omega$

Problem of detection time and place

- $\Omega \subset \mathbb{R}^d$, $\psi_0 \in L^2(\Omega, \mathbb{C}^k)$, detecting surface $\partial\Omega$
- outcome $Z = (T, \boldsymbol{X})$, or $Z = \infty$ if no detector ever clicks
- Problem: Compute the distribution of Z from ψ_0.

Lack of orthodox answer

- Orthodox quantum mechanics (OQM) does not provide a self-adjoint time operator that could be the observable for T.
- Pauli had an argument of the following kind: A time operator $\hat{T}$ would have to be conjugate to the energy operator $\hat{H}$, $[\hat{H}, \hat{T}] = i\hbar$, but that is impossible if the spectrum of $\hat{H}$ is bounded from below.

Aharonov and Bohm's [1961] proposal

for $\Omega = (-\infty, 0] \subset \mathbb{R}^1$, $\partial\Omega = \{0\}$

- Note that for a classical particle with initital position $x < 0$, initial momentum $p > 0$, and mass $m > 0$, the arrival time at the origin is $T = -mx/p$.
- Guess self-adjoint observable $\hat{T} = -\frac{m}{2}\hat{X}\hat{P}^{-1} - \frac{m}{2}\hat{P}^{-1}\hat{X}$.

Not believable

Why not $-m\hat{P}^{-1/2}\hat{X}\hat{P}^{-1/2}$ or $m(-\hat{X})^{1/2}\hat{P}^{-1}(-\hat{X})^{1/2}$?
What about higher dimension, arbitrary surfaces $\partial\Omega$, detection place $\boldsymbol{X}$?
Not derived from more fundamental laws.

Quantum Zeno effect [Turing c. 1950?, Misra and Sudarshan 1977]

- Say, $\Omega = (-\infty, 0] \subset \mathbb{R}^1$ and $\partial\Omega = \{0\}$.
- Let the particle move in $\mathbb{R}^1$.
- Make an instantaneous quantum measurement of the event $x > 0$ (the projection operator $1_{x>0}$) at regular time intervals $\tau > 0$.
- Consider the limit $\tau \to 0$.
- Result: In the limit, the probability of ever finding $x > 0$ becomes 0.
- That seems to make any concept of hard detector impossible. ("A watched pot never boils.")
- Here, a hard detector means one that detects the particle as soon as it arrives at $\partial\Omega$; a soft detector takes a while to notice the particle.
- Yet, it turns out that even a hard detector is possible.

Allcock's [1969] difficulty

- Again, $\Omega = (-\infty, 0]$ and $\partial\Omega = \{0\}$.
- Model of soft detector:
- Consider Schrödinger equation in $\mathbb{R}^1$ with complex potential

$$V(\boldsymbol{x}) = \begin{cases} -iv & \text{if } x > 0 \\ 0 & \text{if } x \leq 0\,, \end{cases}$$

  where $v > 0$ is a constant.
- This means that in the right half line the particle has rate $2v/\hbar$ of being absorbed (loss of $\|\psi\|^2$). $\mathrm{Prob}(X \in dx|T) = |\psi(T,x)|^2 dx$. Average lifetime in the detector volume $= \hbar/2v$.
- Difficulty: In the hard limit $v \to \infty$, $\psi(t,x) = 0$ for $x > 0$ and all $t > 0$, so the particle never gets detected.
- Again, a hard detector seems impossible.

POVMs

- POVM $=$ positive-operator-valued measure
- is a generalization of the concept of observable.
- A self-adjoint operator as observable provides for every possible outcome z a projection E_z.
- A POVM provides for every possible outcome z a positive operator E_z.
- Born rule: $\mathrm{Prob}(Z = z) = \langle\psi|E_z|\psi\rangle$

Def

discrete: $E_z \geq 0$, $\sum_{z\in\mathscr{Z}} E_z = I$.
general (including continuous): $E(B) \geq 0$ for sets $B \subseteq \mathscr{Z}$, $E(\mathscr{Z}) = I$,
$E(B_1 \cup B_2 \cup \ldots) = E(B_1) + E(B_2) + \ldots$ for mutually disjoint B_i.

Let $\mathbb{S}(\mathscr{H}) := \{\psi \in \mathscr{H} : \|\psi\| = 1\}$ be the unit sphere.

Main theorem about POVMs

For every experiment that can be done on a system with arbitrary $\psi \in \mathbb{S}(\mathscr{H})$, there exists a unique POVM E such that the distribution of the outcome Z is

$$\mathrm{Prob}_\psi(Z = z) = \langle\psi|E_z|\psi\rangle \quad \forall z\,.$$

- It's a theorem in Bohmian mechanics (BM).
- OQM is too vague for a precise analysis of measurements, but this statement would count as correct also in OQM.

In fact, $E_z = \langle\phi|U_t^\dagger P(B_z) U_t|\phi\rangle$

with ϕ the apparatus ready state, t the duration of the experiment, U_t the time evolution of system and apparatus, P the position POVM, B_z the set of configurations with the apparatus displaying outcome z, and $\langle\cdot|\cdot\rangle$ the partial inner product in the apparatus variables.

The ideal detector hypothesis

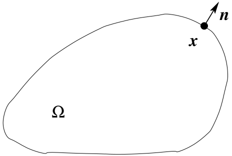
- While the correct POVM $E(\cdot)$ will depend on all details of the detectors, including their quantum states at time 0, the hope is that there is a particular POVM E_0 (or maybe E_κ depending on one or few parameters κ) in the cloud of E's that is a good approximation and can be expressed by some simple rule ("ideal detector hypothesis").
- The hope is nourished by two facts:
  - In practice, detection probabilities do not seem to depend much on the detailed states of the detectors (except that different types of detectors are sensitive to different particle species and at different energy ranges).
  - For detection at a single time t, the distribution of $\boldsymbol{X}$ is $|\psi|^2$, independently of the details of the detector.
- I have a proposal for such a POVM E_κ.
  http://arxiv.org/abs/1601.03715

My proposal: the "absorbing boundary rule"

- Solve the 1-particle Schrödinger equation $i\hbar\dfrac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$ with "absorbing boundary condition" (ABC)

  $$\boldsymbol{n}(\boldsymbol{x})\cdot\nabla\psi(\boldsymbol{x}) = i\kappa\,\psi(\boldsymbol{x})$$

  at every $\boldsymbol{x}\in\partial\Omega$, where $\boldsymbol{n}(\boldsymbol{x}) =$ outward unit normal vector to $\partial\Omega$ at $\boldsymbol{x}$, and $\kappa > 0$ a constant.



- ABC implies that the probability current $\boldsymbol{j}^{\psi} = \frac{\hbar}{m}\mathrm{Im}[\psi^*\nabla\psi]$ points outward at $\partial\Omega$:

  $$\boldsymbol{n}\cdot\boldsymbol{j} = \frac{\hbar}{m}\mathrm{Im}[\psi^*\boldsymbol{n}\cdot\nabla\psi] = \frac{\hbar}{m}\mathrm{Im}[\psi^* i\kappa\psi] = \frac{\hbar}{m}\kappa|\psi|^2 \geq 0\,.$$

- $\mathrm{Prob}_{\psi_0}\Big(T\in dt, \boldsymbol{X}\in d^2\boldsymbol{x}\Big) = \boldsymbol{n}(\boldsymbol{x})\cdot\boldsymbol{j}^{\psi_t}(\boldsymbol{x})\,dt\,d^2\boldsymbol{x}$ assuming $\|\psi_0\| = 1$. $\exists$ POVM E on $\partial\Omega$.
- If the experiments get interrupted at time t before detection, the collapsed wave function is $\psi_t/\|\psi_t\|$.

Another proposal: the naive Bohmian rule

The rule

- Consider $\Omega \subset \mathbb{R}^d$, ψ_0 with support in Ω, $\psi(t)$ evolving freely in $\mathbb{R}^d$.
- Consider Bohmian trajectory $\boldsymbol{Q}(t)$ in $\mathbb{R}^d$ with $\boldsymbol{Q}(0) \sim |\psi_0|^2$,

$$\frac{d\boldsymbol{Q}(t)}{dt} = \frac{\boldsymbol{j}^{\psi(t)}(\boldsymbol{Q}(t))}{|\psi(t,\boldsymbol{Q}(t))|^2}\ .$$

- Consider first arrival time $\tau := \inf\{t \geq 0 : \boldsymbol{Q}(t) \in \partial\Omega\}$.
- Claim: $T = \tau$, $\boldsymbol{X} = \boldsymbol{Q}(\tau)$.

- Plausible in the scattering (far-field) regime "$\partial\Omega \to \infty$."
- The question under which conditions on Ω and ψ_0 this is valid was raised in [Daumer, Dürr, Goldstein, Zanghì 1997].
- This rule was considered in particular by
  - Grübl, Kreidl, and Ruggenthaler [2005]
  - Vona, Hinrichs, and Dürr [2013]
  - Das and Dürr [2019], who assumed that it is generally approximately valid for a spin-$\frac{1}{2}$ particle.

I will explain why Das and Dürr were wrong about this point.

Arrival time vs detection time

- The presence of detectors can change the particle's wave function, and thus the Bohmian trajectories, and thus τ.
- Specifically, in both BM and OQM, a detector can collapse the particle's wave function (even if it doesn't click). If detectors on $\partial\Omega$ have collapsed away parts of ψ that crossed $\partial\Omega$, then these parts cannot propagate back through $\partial\Omega$ (as they partially would have under the free evolution). Thus, even ψ inside Ω is different in the presence and in the absence of detectors.
- So, $\boldsymbol{Q}_{WID}(t) \neq \boldsymbol{Q}_{WOD}(t)$ in general
(WID = with detectors, WOD = without detectors);
so, $T_{WID} \neq T_{WOD} = \tau$ in general.
- The naive Bohmian rule didn't take the presence of detectors into account. There is no reason why the detection time $T = T_D$ should agree with T_{WOD}.

> The Bohmian arrival time is not the Bohmian detection time.

Das and Dürr assumed $T_D = T_{WOD}$.

Consequences of the naive Bohmian rule

- BM would apparently make a prediction that OQM can't make. So one could do an experiment to test BM against OQM. That would be pleasant: Instead of endless debates, do the experiment and every physicist has to agree which theory is right.
- However, that is too good to be true: there is a general argument that BM and OQM are empirically equivalent.
- Moreover, the naive Bohmian rule, if valid, could be used for superluminal signaling.
- However, there is a general argument that superluminal signaling is impossible in BM.

Setup of Das and Dürr

- $\psi : \mathbb{R}_t \times \mathbb{R}^3_{\boldsymbol{q}} \to \mathbb{C}^2$, $\quad i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$
- $\boldsymbol{j}^\psi = \frac{\hbar}{m}\mathrm{Im}[\psi^\dagger \nabla \psi] + \lambda \frac{\hbar}{2m} \nabla \times (\psi^\dagger \boldsymbol{\sigma} \psi)$
- with $\lambda = 1$ (which arises as the non-rel. limit of BM for the Dirac eq)
- real-valued potential V given by (wave guide)

$$V(x,y,z) \equiv \begin{cases} \infty & \text{if } z < 0 \\ \frac{m}{2}\omega^2(x^2+y^2) & \text{if } z \geq 0 \,. \end{cases}$$

- For $\boldsymbol{n} \in \mathbb{S}(\mathbb{R}^3)$, let $|\boldsymbol{n}\rangle$ be the spinor (unique up to phase θ) with $\langle \boldsymbol{n}|\boldsymbol{\sigma}|\boldsymbol{n}\rangle = \boldsymbol{n}$. In spherical coordinates α, β,

$$|\boldsymbol{n}\rangle = e^{i\theta} \begin{pmatrix} \cos(\alpha/2) \\ \sin(\alpha/2)e^{-i\beta} \end{pmatrix} .$$

- Let initial wave function factorize according to $\psi_0 = |\boldsymbol{n}\rangle \otimes \varphi_0$. Fix spatial part $\varphi_0(\boldsymbol{q})$, keep $\boldsymbol{n}$ variable.
- $\Omega = \{z \leq L\}$, $\partial\Omega = \{z = L\}$ for some constant $L > 0$.

Their findings

Das and Dürr computed the distribution of T_{WOD}:

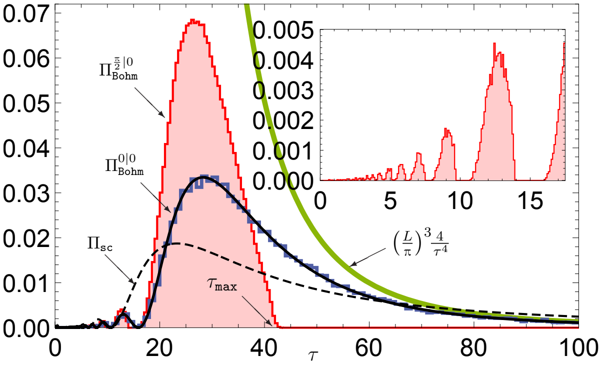


Figure reproduced from their paper. Blue curve: $\alpha = 0$, $\boldsymbol{n} = (0, 0, 1)$.
Red curve: $\alpha = \pi/2$, $\beta = 0$, $\boldsymbol{n} = (1, 0, 0)$.

The distribution depends on $\boldsymbol{n}$.

Decoupling argument

to the effect that the distribution of T_D should not depend on $\boldsymbol{n}$:

- Suppose the Hamiltonian of the particle doesn't couple to the spin, $I_2 \otimes H_0$.
- Suppose the interaction Hamiltonian between particle and detectors doesn't couple to the particle's spin, $I_2 \otimes H_{\rm int}$.
- Then the wave function Φ_t of particle + detectors will factorize $|\boldsymbol{n}\rangle \otimes$ something.
- Thus, the $|\Phi|^2$ distribution of the apparatus display will be independent of $\boldsymbol{n}$, and thus also the distribution of T_D.

However, the first supposition will not be exactly valid for the Dirac Hamiltonian, so I will give another argument.

Distribution of T_{WOD} is not given by a POVM

- If it were given by a POVM $\tilde{E}$, form the spin POVM $E(dt) = \langle \varphi_0 | \tilde{E}(dt) | \varphi_0 \rangle$ acting on $\mathbb{C}^2$, so $\mathrm{Prob}(T \in dt) = \langle \boldsymbol{n} | E(dt) | \boldsymbol{n} \rangle$ for $T = T_{WOD}$.
- Thus, $\mathbb{E}T = \langle \boldsymbol{n} | M | \boldsymbol{n} \rangle$ with $M = \int_0^\infty t\, E(dt)$ a self-adjoint 2×2 matrix.
- Every self-adjoint 2×2 matrix is of the form $M = m_0 I_2 + \boldsymbol{m} \cdot \boldsymbol{\sigma}$ with $m_0 \in \mathbb{R}$, $\boldsymbol{m} \in \mathbb{R}^3$.
- Thus, $\mathbb{E}T = m_0 + \boldsymbol{m} \cdot \boldsymbol{n}$. In particular, if we fix $\beta = 0$ and vary α, so $\boldsymbol{n} = (\sin\alpha, 0, \cos\alpha)$, then

$$\begin{aligned} \mathbb{E}T &= m_0 + m_1 \sin\alpha + m_3 \cos\alpha \\ &= m_0 + \mu \sin(\alpha - \alpha_0)\,, \end{aligned}$$

a shifted sine curve.

α-dependence of $\mathbb{E}\,T_{WOD}$ is not sinusoidal

Plot of α-dependence of $\mathbb{E}\,T_{WOD}$: black or red curve

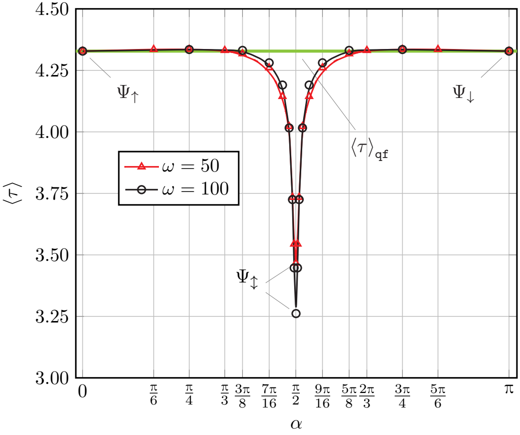


Figure reproduced from [Das and Dürr 2019].
Not a shifted sine curve.

So what has been shown?

- The naive Bohmian rule is not the prediction of BM for a detection time experiment.
- T_{WOD} is in general not measurable.
- Whether T_{WID} is measurable, and whether realistic detectors measure T_{WID}, are largely open questions. The absorbing boundary rule (ABR) suggests yes. This is also supported by certain possible ways of deriving the ABR from BM or OQM outlined by
  - a 2020 paper by Varun Dubey, Cedric Bernardin, and Abhishek Dhar
  - my `http://arxiv.org/abs/2310.01343`

Another example of something not measurable: ψ

What would orthodox quantum physicists predict?

- Not easy to answer, as it is not clear what should be taken as the definition of OQM.
- In practice, OQM often makes use of analogies and quantization.
- Orthodox physicists have not agreed yet on how to compute the distribution of T_D.
- Bohmians would solve the Schrödinger equation for the wave function Φ of the big system formed by the particle, all detectors, a clock, and a recording device, constructed so as to keep a record of which detector clicked when. At a late time t, make a quantum measurement of the record. In OQM, this reasoning would run into the quantum measurement problem.
- But in the end, if it can be shown that the $|\Phi_t|^2$ distribution yields a particular distribution ρ of the record, then orthodox physicists would agree that ρ is the outcome distribution. This argument would trump analogies and guesses. So in the end, OQM agrees with BM.

Different equations of motion

- Several eq.s of motion make the $|\psi|^2$ distribution equivariant, e.g.,
  - see above with different $\lambda \in \mathbb{R}$
  - Nelson's stochastic mechanics
  - Deotto and Ghirardi [1998] gave alternative ODEs (not convincing as laws of nature but mathematically possible)
  - "zig-zag process" for Dirac particles with further hidden variable "handedness" $\in \{+1, -1\}$ [Colin, Wiseman 2011; Struyve 2012; Maes, Meerts, Struyve 2022]
- lead to same distribution of outcome displayed by apparatus
- but lead to different trajectories and thus different T_{WOD}.
- Thus, in most of these theories, $T_D \neq T_{WOD}$.
- That is another reason why the assumption of Das and Dürr was questionable from the start.

Thank you for your attention