Relativistic Collapse Theory With Interaction

Roderich Tumulka

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

November 21, 2019

Plan

- Non-relativistic GRW model (1986)
- Relativistic GRW model for non-interacting particles (2004)
- Relativistic GRW model for interacting particles (2019)
    - Difficulties
    - A simple case first: two flashes
    - The general case
    - Properties

Non-relativistic GRW model

Spontaneous collapse: GRW theory

Key idea:

The Schrödinger equation is only an approximation, valid for systems with few particles ($N < 10^4$) but not for macroscopic systems ($N > 10^{23}$). The true evolution law for the wave function is non-linear and stochastic (i.e., inherently random) and avoids superpositions (such as Schrödinger's cat) of macroscopically different contributions.

Put differently, regard the collapse of ψ as a physical process governed by mathematical laws.

GianCarlo Ghirardi (1935–2018)

Explicit equations by Ghirardi, Rimini, and Weber (1986), Bell (1987)

The predictions of the GRW theory deviate very very slightly from the quantum formalism. At present, no experimental test is possible.

GRW's stochastic evolution for ψ

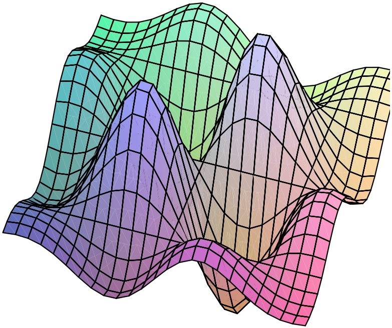
- is designed for non-relativistic quantum mechanics of N particles
- meant to replace Schrödinger eq as a fundamental law of nature
- involves two new constants of nature:
  - $\lambda \approx 10^{-16}\,\text{sec}^{-1}$, called collapse rate per particle.
  - $\sigma \approx 10^{-7}\,\text{m}$, called collapse width.
- Def: ψ evolves as if an observer outside the universe made, at random times with rate $N\lambda$, quantum measurements of the position observable of a randomly selected particle with inaccuracy σ.
- "rate $N\lambda$" means that waiting time $\sim \text{Exp}(N\lambda)$ or $\mathbb{P}(\text{an event in the next } dt \text{ seconds}) = N\lambda\, dt$. [Poisson process]
- more explicitly: Schrödinger evolution interrupted by jumps of the form

$$\psi_{T+} = e^{-\frac{(\boldsymbol{q}_k - \boldsymbol{Q})^2}{4\sigma^2}} \psi_{T-} \,,$$

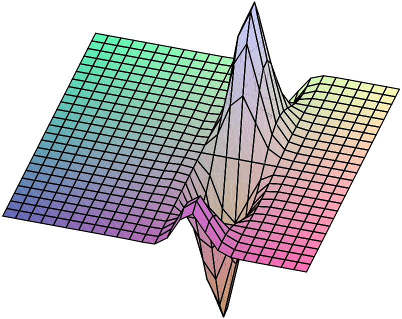
  i.e., multiplication by a Gauss function with random label k, center $\boldsymbol{Q}$ and time T.
- $\mathbb{P}(\boldsymbol{Q} \in d^3\boldsymbol{q}) = \|\psi_{T+}\|^2 d^3\boldsymbol{q} = |\psi_{T-}(\boldsymbol{q}_k = \boldsymbol{q})|^2 * \text{Gaussian}$

GRW's spontaneous collapse

before the spontaneous collapse:



and after:



- In Hilbert space: piecewise deterministic stochastic jump process. ψ_t jumps at random times to random destinations.
- For a single particle, one collapse every 100 million years.
- For 10^4 particles, one collapse every 10,000 years.
- For 10^{23} particles, one collapse every 10^{-7} seconds.
- No-signaling theorem

- As soon as a collapse occurs for one particle in the apparatus, the superposition in the test particle is gone as well.
- A macroscopic superposition $\sum_i \psi_i$ such as Schrödinger's cat would collapse within 10^{-7} seconds.
- It would collapse, up to tails of the Gaussian, to one of the macroscopically distinct wave packets ψ_i (to either $|\text{dead}\rangle$ or $|\text{alive}\rangle$).
- The probability that ψ collapses to ψ_i is, up to Gaussian tails, given by $\|\psi_i\|^2$.
- That is why GRW theory agrees with the standard quantum prediction to an excellent degree of approximation.

Laws for the primitive ontology

Def: GRWf [Bell 1987]

If ψ collapses at time T with center $\boldsymbol{Q}$ then put a flash at $(T, \boldsymbol{Q})$.

Def: GRWm [Diósi 1989; Ghirardi, Grassi, Benatti 1995; Goldstein 1998]

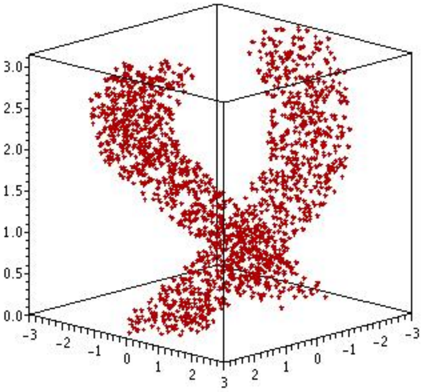
matter is continuously distributed with density given by

$$m(t,\boldsymbol{q}) = \sum_{k=1}^{N} m_k \int \delta^3(\boldsymbol{q}-\boldsymbol{q}_k)\, |\psi_t(\boldsymbol{q}_1,\ldots,\boldsymbol{q}_N)|^2 \, d^3\boldsymbol{q}_1 \cdots d^3\boldsymbol{q}_N$$
$$= \langle \psi_t | \mathcal{M}(\boldsymbol{x}) | \psi_t \rangle$$

with $\mathcal{M}(\boldsymbol{x}) = \sum_{k=1}^{N} m_k\, \delta^3(\boldsymbol{x} - \hat{\boldsymbol{Q}}_k)$ the mass density operators.

GRWf and GRWm are empirically equivalent.

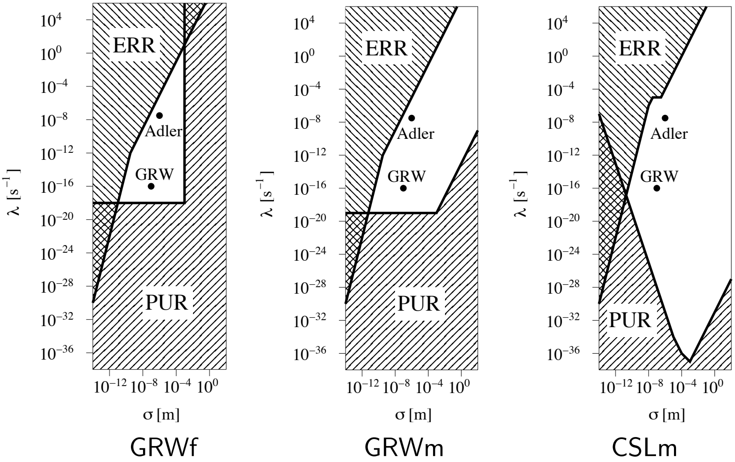
Flash ontology



Instead of particle world lines, there are world points in space-time, called "flashes." A macroscopic object consists of a galaxy of flashes.

GRW theories are empirically adequate

Their predictions deviate very very slightly from the quantum formalism.



Parameter diagrams (log-log scale). ERR = empirically refuted region, PUR = philosophically unsatisfactory region [Feldmann, Tumulka 1109.6579]

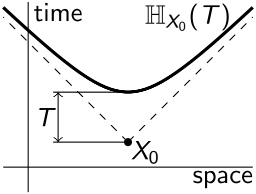
Relativistic GRW model for non-interacting particles (rGRW, 2004)

[Tumulka quant-ph/0406094, quant-ph/0602208, 0711.0035]

- fixed number N of distinguishable particles
- works also in curved space-time, described here in Minkowski space-time $\mathbb{M} = \mathbb{R}^4$
- works also with matter density ontology [Bedingham et al. 1111.1425], described here with flash ontology
- unitary part of evolution: e.g., free Dirac [arising from $L^2(\mathbb{R}^3, \mathbb{C}^4)$]
- with every Cauchy surface Σ there is associated a Hilbert space $\mathscr{H}_\Sigma$
- easier without interaction b/c
  - $U_\Sigma^{\Sigma'} = \otimes_i U_{i\Sigma}^{\Sigma'}$,
  - so propagators for different particles commute,
  - and we can evolve different particles to different surfaces
- need Σ = Cauchy surface or hyperboloid
- assume $U_\Sigma^{\Sigma'}$ exists also for hyperboloids (known for free Dirac with $m > 0$ [Dürr, Pickl math-ph/0207010])

The rGRW process for $N = 1$

<u>Given:</u> initial wave fct ψ_0 on some 3-surface Σ_0, seed flash $X_0 \in \mathbb{M}$



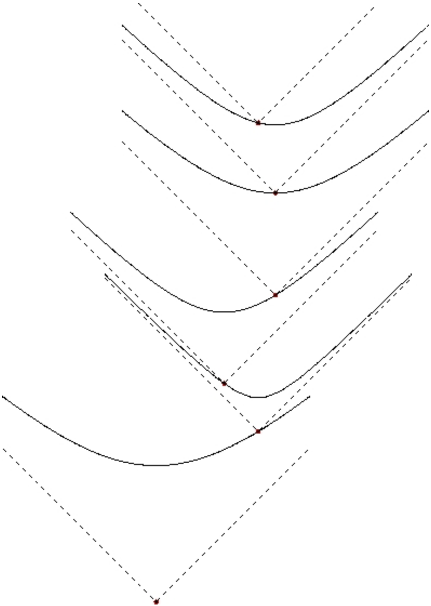
Randomly select next flash $X \in \mathbb{M}$:

- Randomly select waiting time $T \sim \text{Exp}(\lambda)$, T = proper time between X_0 and X, i.e., $X \in \mathbb{H}_{X_0}(T)$
- Evolve $\psi_0 \to \psi_\Sigma$ from Σ_0 to $\Sigma = \mathbb{H}_{X_0}(T)$.
- Randomly select $X \in \Sigma$ with probability density $|\psi_\Sigma|^2 * g$, where $*$ = convolution and g the Gaussian on Σ

$$g(z) = \mathcal{N} \exp\Big(-\frac{\text{dist}_\Sigma(x,z)^2}{2\sigma^2}\Big),$$

$\text{dist}_\Sigma(x,z)$ = spacelike dist. from x to z along Σ, normalization $\int_\Sigma d^3x\, g_x(z) = 1$.

The rGRW process for $N = 1$



Repeat with

ψ_0 replaced by $\dfrac{g_X \psi_\Sigma}{\|g_X \psi_\Sigma\|}$

and X_0 by X.

The rGRW process for $N = 1$

It follows from the definition that the joint distribution of the first n flashes is of the form

$$\mathbb{P}\Big((X_1, \ldots, X_n) \in B\Big) = \langle \psi_0 | G(B) | \psi_0 \rangle, \qquad B \subseteq (\mathbb{R}^4)^n$$

where $\psi_0 \in L^2(\Sigma_0)$, and G is a positive-operator-valued measure (POVM).

The rGRW process for $N > 1$

Let the joint probability distribution of the first n_1 centers for particle 1, ..., the first n_N flashes for particle N be

$$\mathbb{P}\Big((X_{11}, \ldots, X_{n_N,N}) \in B\Big) = \langle \psi_0 | G^{(N)}(B) | \psi_0 \rangle, \quad B \subseteq (\mathbb{R}^4)^{n_1 + \ldots + n_N}$$

where $\psi_0 \in L^2(\Sigma_0)^{\otimes N}$, and $G^{(N)}$ is the product POVM defined by

$$G^{(N)}(B_1 \times \cdots \times B_N) = G(B_1) \otimes \cdots \otimes G(B_N).$$

Explicit form of distribution

- $X_{ik} \in \mathbb{M}$ is the k-th flash of i-th particle
- $\mathbb{H}_{ik} := \mathbb{H}_{X_{ik-1}}(X_{ik}) := \mathbb{H}_{X_{ik-1}}(|X_{ik} - X_{ik-1}|)$
- consider n_i flashes for i-th particle, set $\nu := n_1 + \ldots + n_N$
- $\underline{X} = (X_{ik} : 1 \leq i \leq N, 1 \leq k \leq n_i)$, $\qquad d\underline{x} = \prod_{i=1}^{N} \prod_{k=1}^{n_i} d^4 x_{ik}$

$$\mathbb{P}(\underline{X} \in d\underline{x}) = \langle \psi_0 | D(\underline{x}) | \psi_0 \rangle \, d\underline{x} \quad \text{with}$$

$$D(\underline{x}) := \left(\lambda^{\nu} \prod_{i=1}^{N} \prod_{k=1}^{n_i} 1_{x_{ik} \in \text{future}(x_{ik-1})} e^{-\lambda |x_{ik} - x_{ik-1}|} \right) L(\underline{x})^{\dagger} \, L(\underline{x})$$

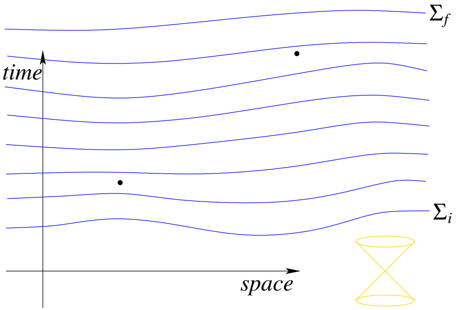
$$L(\underline{x}) := \bigotimes_{i=1}^{N} \prod_{k=1}^{n_i} K(x_{ik}), \qquad K(x_{ik}) := U^{0}_{i\mathbb{H}_{ik}} \, P(g_{x_{ik-1}x_{ik}}) \, U^{\mathbb{H}_{ik}}_{i0}$$

P = multiplication operator, g_{yx} = Gaussian centered at $x \in \mathbb{H}_y(s)$

- Key fact: $\int_{\mathbb{M}^{\nu}} d\underline{x} \, D(\underline{x}) = I$

ψ_Σ

- We have defined the joint distribution of the flashes.
- random wave function ψ_Σ:
- If the flashes X_{ik} up to Σ are given, ψ_Σ is determined by the initial $\psi_0 \in \mathscr{H}_{\Sigma_0}$: Roughly speaking, collapse ψ at every flash and evolve ψ unitarily in-between.



Relativistic GRW model for interacting particles (2019)

Interacting rGRW model

- fixed number N of distinguishable particles
- works also in curved space-time, described here in Minkowski space-time $\mathbb{M} = \mathbb{R}^4$
- works also with matter density ontology [Bedingham et al. 1111.1425], described here with flash ontology
- still want the form $\mathbb{P}\big(\underline{X} \in d\underline{x}\big) = \langle\psi_0|D(\underline{x})|\psi_0\rangle \, d\underline{x}$
- still need to make sure that $\displaystyle\int_{\mathbb{M}^\nu} d\underline{x} \, D(\underline{x}) = I$
- non-relativistic limit = known GRW with interaction
- non-interacting case ≈ known 2004 model
- regard the unitary part $U_\Sigma^{\Sigma'}$ as given and including the interaction

Difficulties

Difficulties (1)

- Guess: still of the form

$$D(\underline{x}) = \left(\lambda^\nu \prod_{i=1}^{N} \prod_{k=1}^{n_i} 1_{x_{ik} \in \text{future}(x_{ik-1})} e^{-\lambda |x_{ik} - x_{ik-1}|}\right) L(\underline{x})^\dagger \, L(\underline{x})$$

- In the non-interacting case,

$$\left(\prod_{ik} \int_{\mathbb{H}_{x_{ik-1}}(s_{ik})} d^3 x_{ik}\right) L(\underline{x})^\dagger L(\underline{x}) = I \, . \qquad (1)$$

This suffices for $\int D = I$ because of the coarea formula

$$\int_{\text{future}(y)} d^4x \, f(x,y) = \int_0^\infty ds \int_{\mathbb{H}_y(s)} d^3x \, f(x,y).$$

Will (1) again be true, or do we need a different strategy?

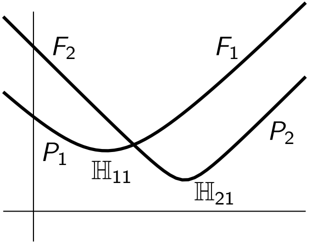
Difficulties (2)

- Guess: something like

$$\text{“}\quad L(\underline{x}) = \prod_{i=1}^{N} \prod_{k=1}^{n_i} U^0_{\mathbb{H}_{ik}} \, P_{\mathbb{H}_{ik}}(g_{x_{ik-1}x_{ik}i}) \, U_0^{\mathbb{H}_{ik}} \quad\text{”} \qquad (2)$$

  with $g_{yxi} = g_{yx}$ in the i-th variable.
- But now $P(g_{ik})$ don't commute for different i. Problem of operator ordering.
- Rough idea:
  - when $x_{j\ell} \in \text{future}(x_{ik})$, put $P(g_{j\ell})$ left of $P(g_{ik})$
  - when $x_{j\ell}$ spacelike from x_{ik}, maybe $P(g_{j\ell})$ commutes with $P(g_{ik})$?
- Don't actually commute because even if $x_{j\ell}$ spacelike from x_{ik}, support$(g_{j\ell})$ is not spacelike from support(g_{ik}).
- Idea: cut off g_{ik} to get better control of support.

Difficulties (3)



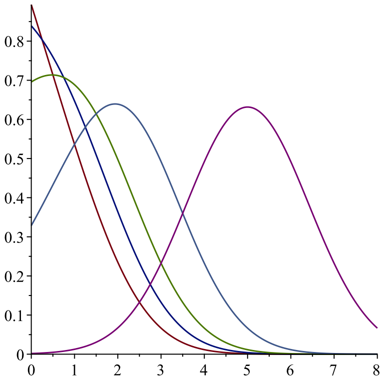
- Previously, $\int_{\mathbb{H}} d^3x\, g_{yx}(z)^2 = 1$.
- Now subdivide $\mathbb{H}_{ik}$ in pieces $=$ past/future of $\mathbb{H}_{j\ell}$.
- For each piece $A \subset \mathbb{H}_{ik}$, define cut-off Gaussian g_A so that $\int_A d^3x\, g_{yAx}(z)^2 = 1_{z\in A}$.

Refined way of cutting off the Gaussian:

$$g_{yAx}(z) := 1_{z\in A}\, 1_{x\in A}\, \|\text{Gaussian}_{yz} 1_A\|^{-1}\, \text{Gaussian}_{yx}(z) \qquad (3)$$

Difficulties (4)

$$g_{yAx}(z) := 1_{z\in A}\, 1_{x\in A}\, \left\| \mathrm{Gaussian}_{yz} 1_A \right\|^{-1} \mathrm{Gaussian}_{yx}(z) \qquad (3)$$

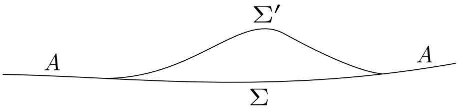


Deviation from Gaussian shape, here on $\mathbb{R}$ instead of $\mathbb{H}$ with $A = [0, \infty)$.

Assumptions

- $U_\Sigma^{\Sigma'} : \mathscr{H}_\Sigma \to \mathscr{H}_{\Sigma'}$
- P_Σ PVM on Σ^N acting on $\mathscr{H}_\Sigma$
- **Interaction locality** (IL): No interaction at spacelike separation.
  Precisely [Lienert, Tumulka 1706.07074],
  For any set $A \subseteq \Sigma \cap \Sigma'$ *in the overlap and any* $i \in \{1, \ldots, N\}$,

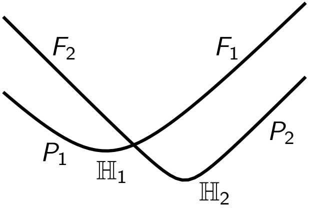
$$P_{\Sigma'}\Big((\Sigma')^{i-1} \times A \times (\Sigma')^{N-i-1}\Big) = U_\Sigma^{\Sigma'} \, P_\Sigma\Big(\Sigma^{i-1} \times A \times \Sigma^{N-i-1}\Big) \, U_{\Sigma'}^{\Sigma}.$$



A simple case first: two flashes

A simple case first: two flashes

$$L(x_1, x_2) := \begin{cases} U^0_{\mathbb{H}_2}\, P_{\mathbb{H}_2}(g_{y_2 P_2 x_2 2})\, U^{\mathbb{H}_2}_{\mathbb{H}_1}\, P_{\mathbb{H}_1}(g_{y_1 P_1 x_1 1})\, U^{\mathbb{H}_1}_0 & \text{if } x_1 \in P_1, x_2 \in P_2 \\ U^0_{\mathbb{H}_2}\, P_{\mathbb{H}_2}(g_{y_2 F_2 x_2 2})\, U^{\mathbb{H}_2}_{\mathbb{H}_1}\, P_{\mathbb{H}_1}(g_{y_1 P_1 x_1 1})\, U^{\mathbb{H}_1}_0 & \text{if } x_1 \in P_1, x_2 \in F_2 \\ U^0_{\mathbb{H}_1}\, P_{\mathbb{H}_1}(g_{y_1 F_1 x_1 1})\, U^{\mathbb{H}_1}_{\mathbb{H}_2}\, P_{\mathbb{H}_2}(g_{y_2 P_2 x_2 2})\, U^{\mathbb{H}_2}_0 & \text{if } x_1 \in F_1, x_2 \in P_2 \\ U^0_{\mathbb{H}_1}\, P_{\mathbb{H}_1}(g_{y_1 F_1 x_1 1})\, U^{\mathbb{H}_1}_{\mathbb{H}_2}\, P_{\mathbb{H}_2}(g_{y_2 F_2 x_2 2})\, U^{\mathbb{H}_2}_0 & \text{if } x_1 \in F_1, x_2 \in F_2. \end{cases}$$



Proposition

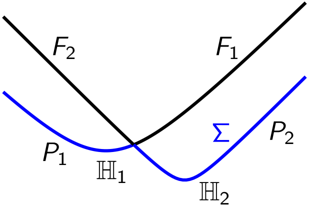
$$\text{(IL)} \Rightarrow \int d^4x_1 \int d^4x_2\, D(x_1, x_2) = I$$

Sketch of proof

It suffices to show that $\int_{\mathbb{H}_1} d^3x_1 \int_{\mathbb{H}_2} d^3x_2\, L(x_1,x_2)^\dagger\, L(x_1,x_2) = I \quad (*)$

Since $\mathbb{H}_i = P_i \cup F_i$,

$\mathbb{H}_1 \times \mathbb{H}_2 = (P_1 \times P_2) \cup (P_1 \times F_2) \cup (F_1 \times P_2) \cup (F_1 \times F_2)$.



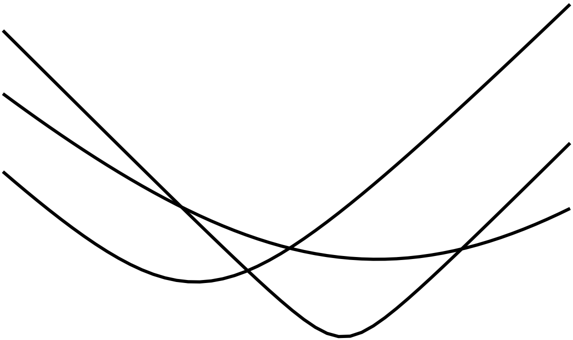
- By $\int_A d^3x\, g_{yAx}(z)^2 = 1_{z\in A}$,
  $\int_A d^3x\, P_{\mathbb{H}}(g_{yAxi})^2 = P_{\mathbb{H}}(1_{x_i\in A})$.
- By (IL),
  $U^{\Sigma}_{\mathbb{H}_2}\, P_{\mathbb{H}_2}(1_{x_2\in P_2})\, U^{\mathbb{H}_2}_{\Sigma} = P_\Sigma(1_{x_2\in P_2})$.
- On the same surface Σ,
  $P_\Sigma(f)$ commutes with $P_\Sigma(g)$.

Put together, calculation ... $\Rightarrow \displaystyle\int_{P_1\times P_2} L^\dagger L = U^0_\Sigma\, P_\Sigma(P_1 \times P_2)\, U^\Sigma_0$.

Similarly for other 3 parts $\Rightarrow$ (*). □

The general case

Division into 4-cells and 3-cells



Admissible sequences

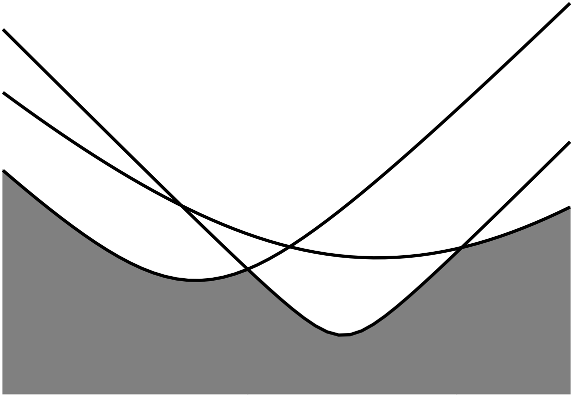
Def: $S \subseteq \mathbb{M}$ is *past complete* $\Leftrightarrow$ $\text{past}(S) \subseteq S$

Fact: $S \neq \mathbb{M}$ past complete iff $S = \text{past}(\partial S)$

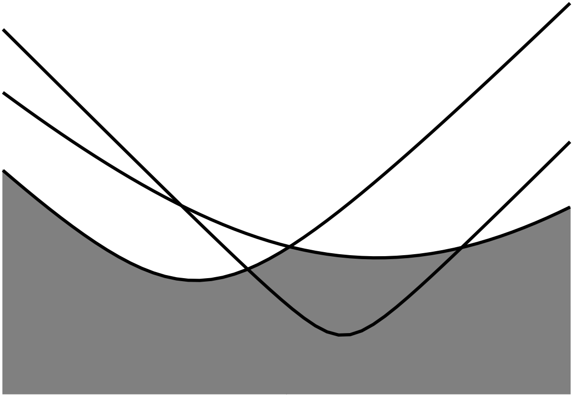
Def: *admissible sequence*: $\big({}^4C_1, \ldots, {}^4C_r\big)$ such that ${}^4C_1 \cup \ldots \cup {}^4C_r = \mathbb{M}$, no repetitions, and for every $n = 1, \ldots, r$, ${}^4C_1 \cup \ldots \cup {}^4C_n$ is past complete.

Proposition: There exist admissible sequences.

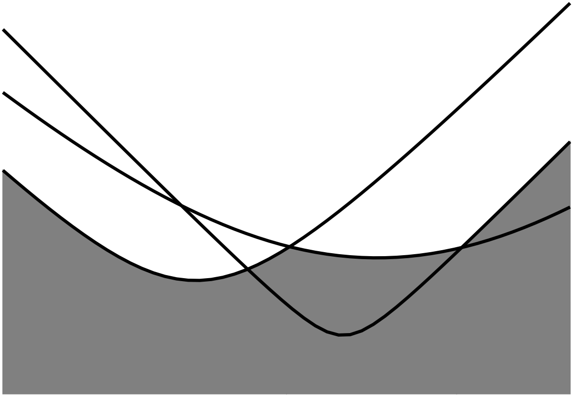
Example of an admissible sequence



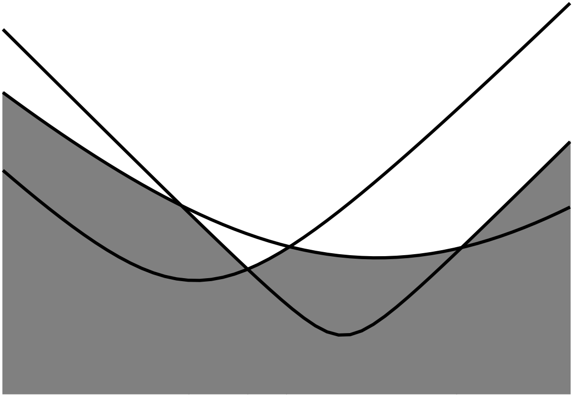
Example of an admissible sequence



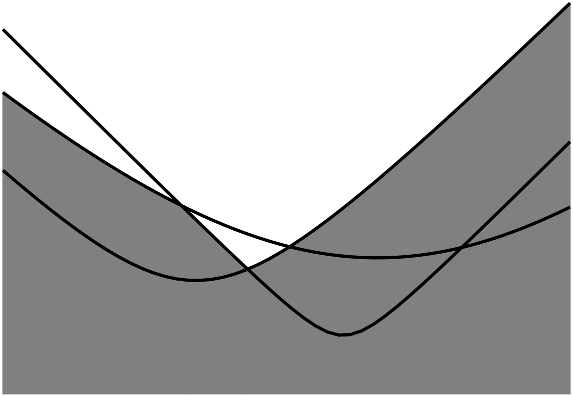
Example of an admissible sequence



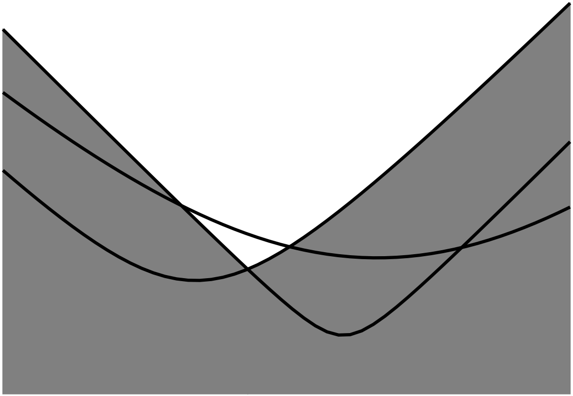
Example of an admissible sequence



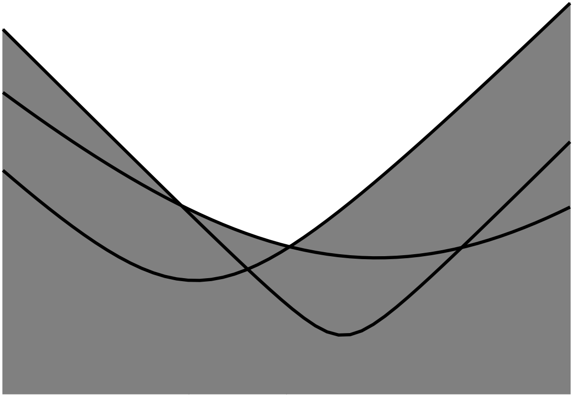
Example of an admissible sequence



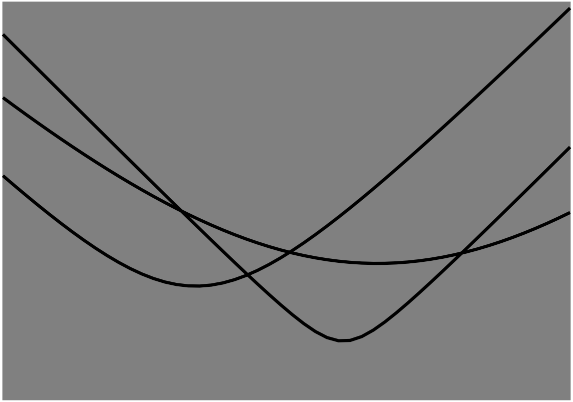
Example of an admissible sequence



Example of an admissible sequence



Example of an admissible sequence



Proposition: Every admissible sequence crosses every 3-cell exactly once.

Since $x_{ik} \in \mathbb{H}_{ik}$, it lies in some 3-cell ${}^3C(x_{ik})$. Set

$$K(x_{ik}) := U^0_{\mathbb{H}_{ik}} \, P_{\mathbb{H}_{ik}}\left(g_{x_{ik-1},{}^3C(x_{ik}),x_{ik},i}\right) \, U_0^{\mathbb{H}_{ik}}.$$

Given an admissible sequence AS, define

$$L(\underline{x}) = \prod_{ik} K(x_{ik})$$

in the order from right to left in which the 3-cells are crossed in AS.

Proposition: When two 3-cells are crossed in the same step, and if (IL) holds, then their K operators commute. Thus, AS unambiguously defines the product $L(\underline{x})$.

Proposition: If (IL) holds, then any two admissible sequences lead to the same operator $L(\underline{x})$. Thus, $L(\underline{x})$ is unambiguously defined.

Key theorem

(IL) $\Rightarrow \int_{\mathbb{M}^\nu} d\underline{x}\, D(\underline{x}) = I$

Sketch of proof of key theorem (1)

(highlights of 5 pages proof) Show that $\left(\prod_{ik} \int_{\mathbb{H}_{x_{ik-1}}(s_{ik})} d^3x_{ik}\right) L(\underline{x})^\dagger L(\underline{x}) = I$.

Fix admissible sequence ${}^4C_1 \ldots {}^4C_r$, count down n from r to 1, set $\Sigma_n = \partial({}^4C_1 \cup \ldots \cup {}^4C_n)$.
By (IL), the projection $P^{ik}_{{}^3C}$ to "$x_{ik} \in {}^3C$" is the same for any surface Σ containing ${}^3C$.
By (IL) again, the projection to the future boundary of ${}^4C$ can be "pulled across" ${}^4C$, i.e., is equal to the projection to its past boundary,

$$P^{ik}_{\partial_+ {}^4C} = P^{ik}_{\partial_- {}^4C} .$$

We know that $\int_{{}^3C} d^3x_{ik}\, K(x_{ik})^\dagger K(x_{ik}) = P^{ik}_{{}^3C}$. To use it, need that x_{ik} is the rightmost integral, and that $K(x_{ik})$ is the leftmost factor in $L(\underline{x})$.

(cont'd)

Sketch of proof of key theorem (2)

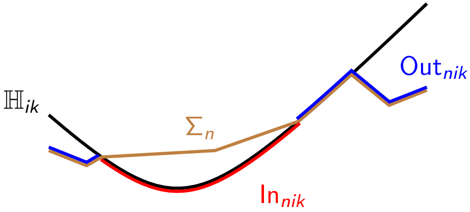
Induction hypothesis

$\int_{\prod \mathbb{H}} d\underline{x}\, L^\dagger L$ equals the sum of all terms of the form

$$\Bigg(\prod_{ik:C(ik)\in \mathrm{In}_{ik}} \int_{C(ik)} d^3x_{ik}\Bigg)\Bigg(\prod_{ik:C(ik)\in \mathrm{In}_{ik}} K(x_{ik})\Bigg)^\dagger \Bigg(\prod_{ik:C(ik)\in \mathrm{Out}_{ik}} P^{ik}_{C(ik)}\Bigg)\Bigg(\prod_{ik:C(ik)\in \mathrm{In}_{ik}} K(x_{ik})\Bigg)$$

with $C(ik) \in \mathrm{In}_{ik} \cup \mathrm{Out}_{ik}$ ("in-cell" = is integrated over, "out-cell" = has already been integrated out), where

$$\mathrm{In}_{ik} = \mathrm{In}_{nik} = \mathbb{H}_{ik} \cap (\mathrm{past}(\Sigma_n) \setminus \Sigma_n)\,, \quad \mathrm{Out}_{ik} = \mathrm{Out}_{nik} = \Sigma_n \cap \mathrm{past}(\mathbb{H}_{ik})\,.$$



Sketch of proof of key theorem (3)

Relation of "which cell borders on which" is independent on the exact location of the $\mathbb{H}_{ik}$ $\Rightarrow$ need to consider cells abstractly.

Induction step:

1. Pull projections on $\partial_+ {}^4C$ across ${}^4C$. (need to combine several summands)
2. In each summand, integrate out x_{ik} if $C(ik) \subseteq \partial_- {}^4C$. Check that no other integral or $P^{j\ell}_{{}^3C}$ depends on x_{ik}. Obtain factor $P^{ik}_{C(ik)}$.

□

Properties

Properties

- Non-local
- Size of 3-cells: back-of-envelope estimate 10^{-3} m (no problem)
- Stochastic evolution of ψ_Σ: similarly as before
- Non-interacting special case $\approx$ 2004 model (exact if we replace cut-off Gaussians by Gaussians; tiny change if size of 3-cell $= 10^4\,\sigma$)
- Microscopic parameter independence (i.e., joint distribution of flashes before Σ is independent of external fields after Σ): holds approximately.
- No superluminal signaling (follows from microscopic parameter independence): holds approximately
- Non-relativistic limit ("$c \to \infty$") $=$ GRW 1986
  hyperboloid $\to$ horizontal 3-plane
  3-cell $\to$ horizontal 3-plane
  4-cell $\to$ layer between horizontal 3-planes
  cut-off Gaussian $\to$ Gaussian
  there is only 1 admissible sequence

Thank you for your attention