On the Present Status of Quantum Mechanics

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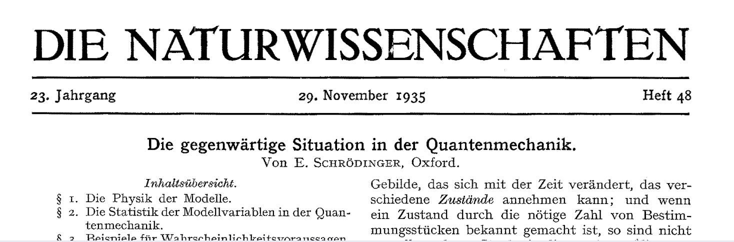

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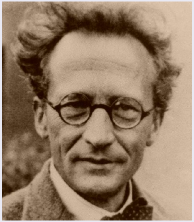
Die gegenwärtige Situation in der Quantenmechanik.

Von E. Schrödinger, Oxford.

Inhaltsübersicht.

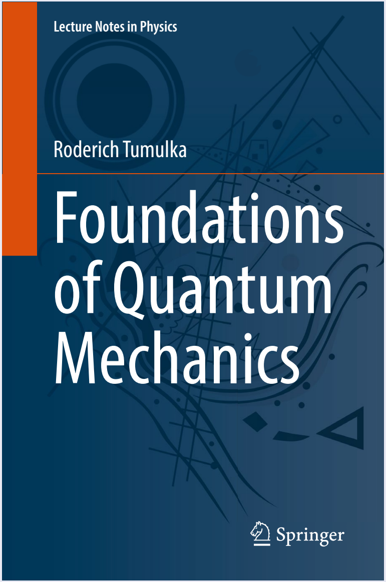
§ 1. Die Physik der Modelle.
§ 2. Die Statistik der Modellvariablen in der Quantenmechanik.
§ 3. Beispiele für Wahrscheinlichkeitsvoraussagen.

Gebilde, das sich mit der Zeit verändert, das verschiedene *Zustände* annehmen kann; und wenn ein Zustand durch die nötige Zahl von Bestimmungsstücken bekannt gemacht ist, so sind nicht



E. Schrödinger (1887–1961)

Advertisement



I'll be teaching a course on Foundations of Quantum Mechanics in the winter semester 2023/24,
and I've written a book about it in 2022.

What the debate on “foundations” is about

Everbody agrees on

rules for empirical predictions: unitary evolution, Born’s rule, collapse rule

What we may want more

We would like to know: how does nature do it, what is the explanation of the observed outcome statistics? What actually happens?

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- and is a technical term in the foundations of QM:
- The ontology of a theory T is what exists in the world according to T.
- Example: The ontology of Newtonian mechanics consists of 3d space, 1d time, and particles with trajectories.

Plan of the talk

- What's the problem?
- 3 known solutions: trajectories, many worlds, spontaneous collapse
- 3 theorems: from 1957, 2012, 2022

What's the problem?

Measurement process

Consider an ideal quantum measurement of the observable $A = \sum_\alpha \alpha P_\alpha$ with eigenvalues α and P_α the projection to the corresponding eigenspace. It begins at t_0 and ends at t_1. At t_0, the wave fct of object and apparatus is

$$\Psi(t_0) = \psi(t_0) \otimes \phi$$

with $\psi(t_0) =$ wave fct of the object, $\phi =$ ready state of the apparatus. By the Schrödinger eq., Ψ evolves to

$$\Psi(t_1) = e^{-iH(t_1 - t_0)} \Psi(t_0) \,.$$

Measurement process, continued

We have that $\Psi(t_0) = \psi(t_0) \otimes \phi$ and $\Psi(t_1) = e^{-iH(t_1-t_0)}\Psi(t_0)$.

Suppose first that the object is in an eigenstate ψ_α of A. Then

$$\Psi_\alpha := \Psi(t_1) = e^{-iH(t_1-t_0)}[\psi_\alpha \otimes \phi]$$

should be a state in which the apparatus displays the value α (e.g., by the position of a needle).

Suppose next that $\psi(t_0) = \sum_\alpha c_\alpha \psi_\alpha$ is an arbitrary superposition. Then

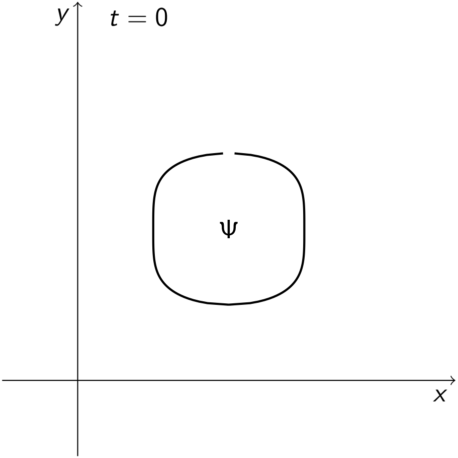
$$\Psi(t_0) = \sum_\alpha c_\alpha \left[\psi_\alpha \otimes \phi\right]$$

and, by linearity of the Schrödinger eq.,

$$\Psi(t_1) = \sum_\alpha c_\alpha \Psi_\alpha \,,$$

i.e., a superposition of wave functions of apparatuses displaying different outcomes.

Evolution of Ψ in configuration space of system x + apparatus y:



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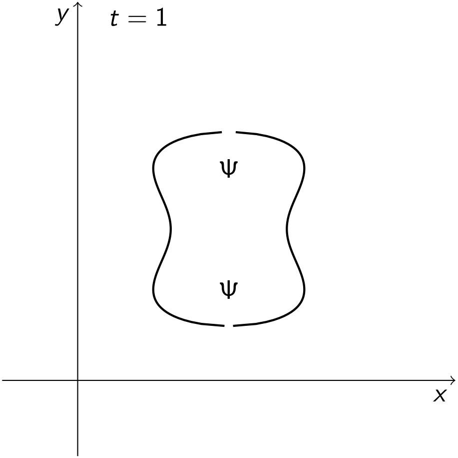

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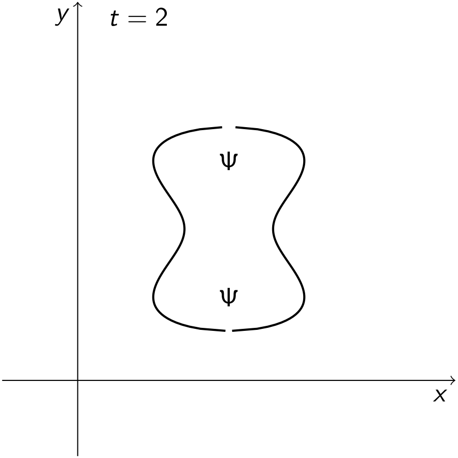

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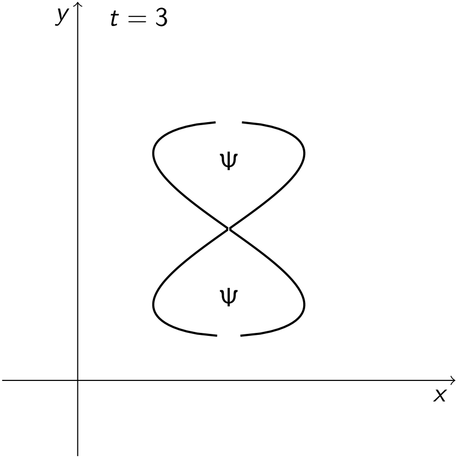

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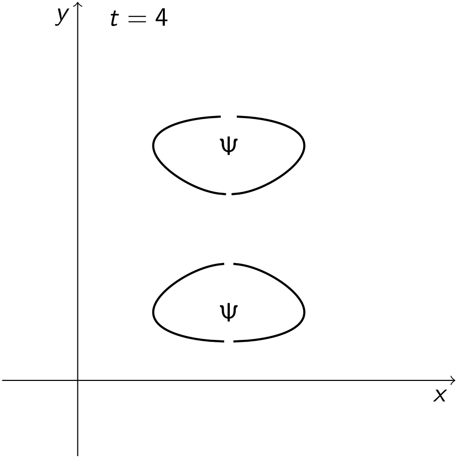

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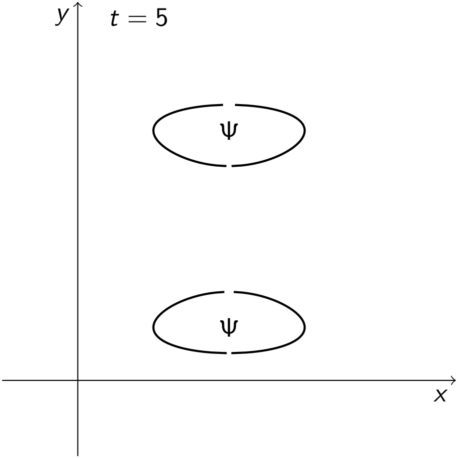

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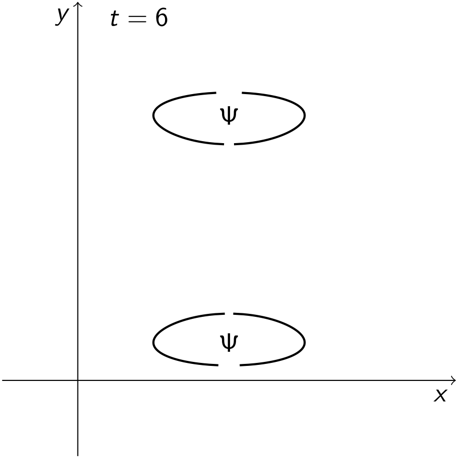


What the problem is

- The apparatus consists of electrons and quarks, so it should be possible to treat it like a quantum system with a wave fct ϕ on $\mathbb{R}^{3N}$, $N > 10^{23}$.
- If we do, then $\Psi(t_0) = \psi \otimes \phi$ evolves according to the Schrödinger eq. to $\Psi(t_1) = \sum_\alpha c_\alpha \Psi_\alpha$, where Ψ_α corresponds to a needle pointing to α. A superposition of different outcomes.
- $\Psi(t_1)$ doesn't say what the actual outcome is.
- We might have expected a state $\Psi(t_1)$ with a unique needle position.
- We might have expected a random state because the outcome should be random.

Schrödinger's cat

is a particular version of the problem. Schrödinger formulated it to criticize the Copenhagen interpretation of QM.

What exactly is the problem?

Bob: Superposition or not, $\Psi(t_1)$ still yields the right probabilities.

Alice: Everybody agrees about the empirically right probabilities. That is not the problem.

Bob: Then where is the problem?

Alice: The problem is about what is there in reality.

Bob: I believe there is no microscopic reality, that observables don't have values before the measurement.

Alice: But that is a hypothesis about reality, too. If only the wave function exists, that is a hypothesis about reality, too. The measurement problem puts constraints on the possible hypotheses about reality.

Bob: Which constraints?

Let's pin down the problem

3 assumptions

1. In each run of the experiment, there is a unique outcome.
2. The wave function is a complete description of a system's physical state in reality. (There are no further variables.)
3. The time evolution of the wave function of an isolated system, not entangled with the outside, is always given by the Schrödinger eq.

Together, they lead to a contradiction: By 3, $\Psi(t_1)$ is generically a superposition of Ψ_α corresponding to different outcomes. Thus, $\Psi(t_1)$ doesn't select an outcome. If there were further variables, they could select an outcome, but by 2 there aren't. Thus, there is no unique outcome, in contradiction to 1.

Consequence

We need to drop one of the 3 assumptions.

Three solutions to the problem

#1: Many worlds

Everett (1957) argued that all contributions to $\Psi(t_1)$ are real, so all possible outcomes are realized "somewhere." (Assumption 1 is denied.)

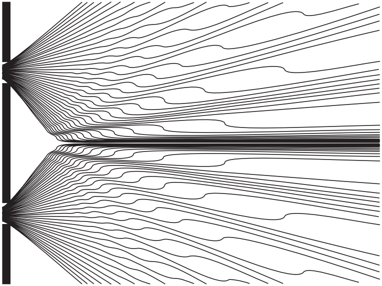
Clearest version of this theory [Schrödinger 1925]:

Law 1: the Schrödinger eq for the wave function Ψ of the universe.
Law 2: matter is distributed in $\mathbb{R}^3$ with density
$m_t(\boldsymbol{x})$ = 3-marginal of $|\psi_t(\boldsymbol{x}_1 \ldots \boldsymbol{x}_N)|^2$.

It seems that ultimately, this kind of theory doesn't work, specifically can't account for probabilities. (But it's subtle: The theory requires that a law of nature determines how to count worlds, and plausibly, a law of nature can't determine that.)

#2: Bohm's trajectories

The obvious trajectories are the flow lines of the probability 4-current $(\rho, j) = (|\psi|^2, \operatorname{Im} \psi^* \nabla \psi)$. Bohm's (1952) proposal is to take them seriously, to hypothesize that electrons are literally point particles.



Drawn by G. Bauer after Philippidis et al.

(Most contemporaries hated that. They had spent years practicing Copenhagen philosophy, and now difficult philosophy might be replaced by a simple equation.) The configuration Q_t is $|\psi_t|^2$-distributed at all t.

It works. (Assumption 2 is denied.)

Inhabitants of a universe governed by Bohmian mechanics would make observations in agreement with the rules of QM.

Consequence

No empirical test between Bohmian mechanics and Copenhagen QM is possible.

So what is a theory like Bohm's good for?

- For understanding
  (cf. Copernicus vs Ptolemy)
- For precise reasoning
  (cf. mathematicians' definitions of the integral)

#3: Spontaneous collapse (GRW theory)

Key idea:

The Schrödinger equation is only an approximation, valid for systems with few particles ($N < 10^4$) but not for macroscopic systems ($N > 10^{23}$). The true evolution law for the wave function is non-linear and stochastic (i.e., inherently random) and avoids superpositions (such as Schrödinger's cat) of macroscopically different contributions.

Put differently, regard the collapse of ψ as a physical process governed by mathematical laws.

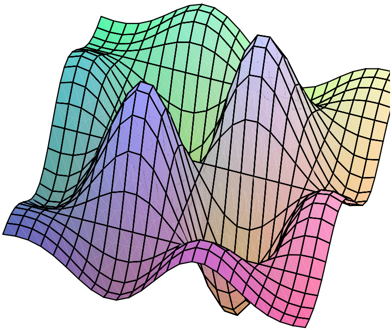
GianCarlo Ghirardi (1935–2018)

Explicit equations by Ghirardi, Rimini, and Weber [Phys.Rev. D 1986]

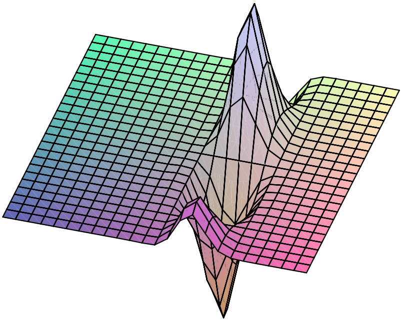
The predictions of the GRW theory deviate very very slightly from the quantum formalism. At present, no experimental test is possible.

GRW's spontaneous collapse

before the "spontaneous collapse":



and after:



Three theorems

Quantum measurements are not literally measurements

The “no-hidden-variables” (NHV) theorem was first proved by Gleason (1957), then in other/simpler ways by Specker (1960), Bell (1966), Kochen and Specker (1967), Mermin (1990), and Peres (1991).

Let us suppose that with every self-adjoint operator A there is associated a physical quantity v_A, the actual value of the observable A, and that a quantum measurement of A simply reveals the value v_A. Can it be this way?

#1: NHV theorem

Suppose $3 \le \dim \mathscr{H} < \infty$. Let $\mathscr{A}$ be the set of all self-adjoint operators on $\mathscr{H}$, and fix a $\psi \in \mathscr{H}$ with $\|\psi\| = 1$. The Born distribution for $A \in \mathscr{A}$ is

$$\text{Prob}(\alpha) = \|P_\alpha \psi\|^2 = \langle \psi | P_\alpha \psi \rangle \quad (1)$$

for $A = \sum_\alpha \alpha P_\alpha$. For pairwise-commuting A, B, C with $A = \sum_{\alpha\beta\gamma} \alpha P_{\alpha\beta\gamma}$, $B = \sum_{\alpha\beta\gamma} \beta P_{\alpha\beta\gamma}$, $C = \sum_{\alpha\beta\gamma} \gamma P_{\alpha\beta\gamma}$, the joint Born distribution is

$$\text{Prob}(\alpha, \beta, \gamma) = \|P_{\alpha\beta\gamma} \psi\|^2 . \quad (2)$$

NHV theorem

Consider a joint distribution of random variables v_A for all $A \in \mathscr{A}$. Suppose that a quantum measurement of any $A \in \mathscr{A}$ yields v_A. Suppose further that whenever $A, B \in \mathscr{A}$ commute, then a quantum measurement of A doesn't change the value of v_B (nor that of v_A). Then the joint distribution of v_A, v_B, v_{A+B} disagrees with the joint Born rule (2).

#2: Wave functions are real

Pusey-Barrett-Rudolph (PBR) theorem (2012)

In *any* theory whose empirical predictions agree with the rules of QM, $\mathbb{C}\psi$ must be part of the ontology or a function of the ontology.

Formalization of "any theory"

Write λ for an "ontic state" (what is real), Λ for the set of all λ's (the "ontic space").

• Given any quantum state ψ, there must be a probability distribution $\varrho^\psi(d\lambda)$ over Λ.

• Given any experiment $\mathscr{E}$, there must be a probability distribution $P_{\lambda,\mathscr{E}}$ for the outcomes if $\mathscr{E}$ is performed on a system in the ontic state λ.

• Empirical agreement $\Leftrightarrow \int_\Lambda \varrho^\psi(d\lambda)\, P_{\lambda,\mathscr{E}}(B) = \langle\psi|E_{\mathscr{E}}(B)|\psi\rangle\ \forall$ set B

Example: in Bohmian mechanics, $\lambda = (Q,\psi)$, $\Lambda = \mathbb{R}^{3N} \times \mathscr{H}$, $\varrho^\psi(dQ\, d\varphi) = |\psi(Q)|^2\, \delta(\psi - \varphi)\, dQ\, d\varphi$.

- Terminology: "every theory" = "ontological model"
- If Bob is "not a realist," then he wants $\lambda = \psi$.

Formalization of "part of the ontology or a function of the ontology"

$\Leftrightarrow$ "$\psi = f(\lambda)$" $\Leftrightarrow$ ϱ^ψ disjoint from ϱ^ϕ whenever $\mathbb{C}\psi \neq \mathbb{C}\phi$

#3: Positivism in QM

Here, "positivism" is the view that

- a statement is unscientific or even meaningless if it can't be tested experimentally
- an object is not real if it can't be observed
- a variable is not well defined if it can't be measured.

Feynman didn't like that:

"Does this mean that my observations become real only when I observe an observer observing something as it happens? This is a horrible viewpoint. Do you seriously entertain the thought that without observer there is no reality?" (1959)

Richard Feynman (1918–1988)

- Positivistic reasoning is often implicit in textbook arguments.
- For example: you can't observe the ether, therefore the ether doesn't exist.
- But you could make the following argument instead: there is a consistent theory without the ether (special relativity) that makes empirically correct predictions, so there is no need to assume the existence of an ether.
- From Bohmian mechanics it follows that it is impossible to measure the instantaneous velocity of a particle unless we already know the particle's wave function. So there is a limitation to knowledge. That runs against positivism.
- In classical mechanics, there are no limitations to knowledge. But in QM ...

A theorem against positivism

Theorem [Tumulka, Found.Phys. 2022, arxiv.org/abs/2205.05520]

In every theory whose empirical predictions agree with the rules of QM, there is a quantity that is well defined but can't be measured.

So, limitations to knowledge are inevitable, and positivism is provably wrong. Just as not all energy contained in Carnot's (1824) heat engines can be extracted as useful work, not all information contained in a quantum system can be extracted as human knowledge. And it is not a flaw of Bohmian mechanics that velocities can't be measured.

Precise statement

Given any ontological model $(\Lambda, \varrho^\psi(d\lambda), P_{\lambda,\mathscr{E}})$ with empirical agreement with the rules of QM for sufficiently many observables (i.e., such that $E_{\mathscr{E}}(B)$ covers at least all 1d projection operators), then there is no experiment $\mathscr{E}$ that can measure λ.

Thank you for your attention

Relativistic space-time

Can Bohmian mechanics and GRW theory be made relativistic?

That depends on what counts as "relativistic."

- no superluminal signaling: both
- foliation independence: GRW yes [Tumulka 2006], Bohm no

So, GRW is "more relativistic" than Bohm. Still, general relativity has special foliations, so one could say that Bohmian mechanics is as relativistic as general relativity.

Thank you for your attention