

Talks on Motives

The category $\mathcal{M}$ of motives is a hypothetical abelian category through which all classical cohomology theories for varieties should factor by an additive functor. In that way, assertions like the Künneth decomposition (and much more) should hold in $\mathcal{M}$ thus yielding Künneth formulas for all cohomology theories at once.

1. **Introduction to motives** **Paul Vögele**
de Rham and singular cohomology and comparison, Grothendieck sites and sheaf cohomology, de Rham in positive characteristic, Witt vectors, lift and crystalline cohomology, Chow ring, correspondences, desired properties of motives, effective motives
[1, 12] and [4, 13, 21]
2. **Tannakian categories** **Giacomo Gavelli**
Tannaka-Krein duality and locally constant sheaves, monoidal categories, tensor categories, main theorem
[5] and [17, 18]
3. **The standard conjectures** **Clemens Nollau**
the Weil conjectures, Weil cohomology, correspondences, Lefschetz and Hodge conjectures, permanence properties
[9], for the last point: [2], general: [22]
4. **Classical motives** **Dan Funck**
rational equivalence, Chow groups, pseudo abelian categories, rational motives, Manin's identity principle, curves and abelian varieties
[19] and [8, 10]
5. **Voevodsky's derived category of motives** **Felix Röhrlé**
[20], see also [3]

More material in [11, 14, 15]

References

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