SWAS Süd-West-Arithmetik-Seminar SS 2010
Chen's iterated integrals and applications

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0. Preparation: This background material is needed in order to follow the program. There will not be an additional meeting to cover this.

- 0.1 Pure Hodge structures [PS] 1. & 2.
- 0.2 Mixed Hodge structures: definition of pure and mixed Hodge structures, abelian category, Hodge decomposition for cohomology of compact Kähler manifolds, mixed Hodge structure on the cohomology of a smooth variety, polarization e.g [PS] §3, 4.1
- 0.3 Cohomology of H-spaces: Theorem of Borel-Serre, [PS] 8.1
- 0.4 Variation of Mixed Hodge structures, admissibility: definition and properties in the compact and non-compact case, Gauss-Manin connection e.g. [PS] §10, [BZ] §7

0.1 - 0.3 are needed for the first meeting, 0.4 for the third.

1. Meeting: (Karlsruhe, Mo 10.5.10) Chen's theory of iterated integrals. The aim is to formulate and prove Chen's results on the de Rham description of cohomology of the path space of complex algebraic varieties. The original reference is [C], see also [BZ] §6, [HZ] §3. I suggest following [PS]. The aim of the first meeting is to show [PS] Theorem. 8.26.

- 1.1 formulation of Chen's results; [PS] 8.2 with main emphasis on fundamental group including complement (I), see also [C], [BZ]. Definition of iterated integrals [PS] 8.3.1
- 1.2 the homotopy de Rham theorem [PS] 8.3.2., Theorem 8.23 with a sketch of proof, see [C]
- 1.3 Iterated integrals and the fundamental group, [PS] 8.4. The main emphasis should be Theorem 8.26 and its proof. see also [C], [BZ].

2. Meeting: (Tübingen, Fr 11.6.10) Chen's theory is used to put a mixed Hodge structure on the completed group ring. The aim of the second meeting is then to show that the mondromoy representation of unipotent variations of mixed Hodge structure is compatible with this Hodge structure and that this fact determines the variation uniquely.

- 2.1 The mixed Hodge structure on the fundamental group, [PS] 8.5, in particular Cor. 8.32, see also [C], [BZ]
- 2.2 Unipotent variations of Hodge structure and the monodromy representation: Define the notion of an admissible unipotent variaton of Hodge structure, [BZ] Def. 7.2, [HZ] §1; formulate and and prove [HZ] Theorem 7.2: the monodromy representation is a morphism of Hodge structures. The discussion of admissibility should not be given much room.

2.3 rigidity: formulate and prove rigidity in [BZ] Theorem 7.12. We are only interested in the simpler case of an algebraic base and a unipotent variation, see also [HZ] §8.

3. Meeting: (Freiburg, Mo 12.7.10) The results are strengthened to a universal property. Admissible unipotent variations of mixed Hodge structure are in 1-1 correspondence with Hodge theoretic representations of the fundamental group. We then look at a couple of exlicit applications.

3.1 The universal property: Formulate [BZ] Theorem 7.19 or [HZ] Theorem 1.6, sketch of proof [HZ] §3-5, remark at end of [HZ] §8

3.2 Construction of logarithm and polylogarithm variations on $\mathbf{P}^1 \setminus \{0, 1, \infty\}$ from the universal property, explicit description and evaluation at roots of unity [HW] III, IV, V.

3.3 Discuss [M] and/or [M2].

References

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[PS] C. Peters, J. Steenbrink, Mixed Hodge structures. Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], 52. Springer-Verlag, Berlin, 2008.

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[HW] Huber, Wildeshaus, The Classical Polylogarithm, Abract of a series of lectures given at the workshop on polylogs in Essen, May 1-4, 1997, available at http://home.mathematik.uni-freiburg.de/arithmetische-geometrie/forschung.html

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