Corrections to: A First Course in Harmonic Analysis, paperback version

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I thank Ghassem Narimani for pointing out an error in the proof of the Riemann-Lebesgue Lemma 3.3.2 on page 48. Replace it with the following proof:

Proof: We compute

$$
\begin{aligned}
f(x) &= \int_{-\infty}^{\infty} f(y)\, e^{-2\pi i x y}\, dy \\
&= -\int_{-\infty}^{\infty} f(y)\, e^{-2\pi i x \left(y+\frac{1}{2x}\right)}\, dy \\
&= -\int_{-\infty}^{\infty} f\left(y-\frac{1}{2x}\right) e^{-2\pi i x y}\, dy.
\end{aligned}
$$

So we get

$$
\hat{f}(x) = \frac{1}{2}\int_{-\infty}^{\infty} \left(f(y) - f\left(y-\frac{1}{2x}\right)\right) e^{-2\pi i x y}\, dy.
$$

Let $\varepsilon > 0$. By the integrability of f there exists $T > 0$ such that $\int_{|x|>T-1} |f(x)|\, dx < \varepsilon/2$. Let

$$
\hat{f}_T(x) = \frac{1}{2}\int_{-T}^{T} \left(f(y) - f\left(y-\frac{1}{2x}\right)\right) e^{-2\pi i x y}\, dy.
$$

For $|1/2x| < 1$ we estimate

$$
\begin{aligned}
&\left|\frac{1}{2}\int_{|y|>T} \left(f(y) - f\left(y-\frac{1}{2x}\right)\right) e^{-2\pi i x y}\, dy\right| \\
&\qquad \le \frac{1}{2}\int_{|y|>T} |f(y)|\, dy + \frac{1}{2}\int_{|y|>T} |f(y-\frac{1}{2x})|\, dy \\
&\qquad\qquad \le \int_{|y|>T-1} |f(y)|\, dy < \varepsilon/2.
\end{aligned}
$$

So if $|x| > \frac{1}{2}$ we get

$$|\hat{f}(x)| < |\hat{f}_T(x)| + \frac{\varepsilon}{2}.$$

By dominated convergence and the continuity of f it follows that there exists x_0 such that for $|x| > x_0$ one has $|\hat{f}_T(x)| < \frac{\varepsilon}{2}$. Together the claim follows. Q.E.D.

(page,location)

(46,*) It is not mentioned, but used, that the Fourier-transform of a function $f \in L^1_{bc}(\mathbb{R})$ is a continuous function. This is an immediate consequence of the dominated convergence theorem.

65,-10 The integration should extend from $-\infty$ to ∞.

(66,Lemma 4.4.1) The proof of the lemma should be:

$$\hat{\hat{T}}(f) = \hat{T}(\hat{f}) = T(\hat{\hat{f}}) = T(\check{f}).$$

(104,-4) Add: From the fact that each $\chi \in \hat{A}$ is continuous, it follows that $\bigcup_n L_n = \hat{A}$.

(106,10) Write: The lemma follows. To finish the proof of the theorem, it remains to show that $\hat{A}$ is locally compact. For this fix a natural number k such that $B_{1/k}$ has compact closure. Since the prescribed exhaustion (K_n) is absorbing, there exists n with $B_{1/k} \subset K_n$. Let $\hat{e}$ be the identity character in $\hat{A}$. Then, for any character $\chi \in \hat{A}$,

$$\hat{d}(\chi, \hat{e}) < \frac{\sqrt{2}}{2^n} \quad \Rightarrow \quad \hat{d}_n(\chi, \hat{e}) < 2^n \frac{\sqrt{2}}{2^n} = \sqrt{2} \quad \Rightarrow \quad \chi \in L_k.$$

Hence, L_k is a neighborhood of $\hat{e}$ in $\hat{A}$. Since L_k is compact, $\hat{A}$ is locally compact.