Problem Set # 1

Due date: Monday, May 4th, 2009 at the beginning of the class.

All the problem are taken from the 2nd edition of the book *An Elementary Introduction to Mathematical Finance* by Sheldon M. Ross.

Exercise 4.9 A friend purchased a new sound system that was selling for $4,200. He agreed to make a down payment of $1,000 and to make 24 monthly payments of $160, beginning one month from the time of purchase. What is the effective interest rate being paid?

Exercise 4.30 Suppose you can borrow money at an annual interest rate of 8% but can save money at an annual interest rate of only 5%. If you start with zero capital and if the yearly cash flows of an investment are

\[-1000, \quad 900, \quad 800, \quad -1200, \quad 700,\]

should you invest?

Exercise 5.1 Suppose it is known that the price of a certain security after one period will be one of the \(n\) values \(s_1, \ldots, s_n\). What should be the cost of an option to purchase the security at time 1 for the price \(K\) when \(K < \min_{i=1}^{n} s_i\)?

Exercise 5.9 A European call and put option on the same security both expire in three months, both have a strike price of 20, and both sell for the price 3. If the nominal continuously compounded interest rate is 10% and the stock price is currently 25, identify an arbitrage.

Exercise 5.16 Let \(S(t)\) be the price of a given security at time \(t\). All of the following options have exercise time \(t\) and, unless stated otherwise, exercise price \(K\). Give the payoff at time \(t\) that is earned by an investor who:

(a) owns one call and one put option;
(b) owns one call having exercise price \(K_1\) and has sold one put having exercise price \(K_2\);
(c) owns two calls and has sold short one share of the security;
(d) owns one share of the security and has sold one call.

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Exercise 5.20 Let $P(K, t)$ denote the cost of a European put option with strike $K$ and expiration time $t$. Prove that $P(K, t)$ is convex in $K$ for fixed $t$, or explain why it is not necessarily true.

Exercise 6.2 Consider an experiment with four possible outcomes, and suppose that the quoted odds for the first three of these outcomes are as follows.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Odds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

What must be the odds against outcome 4 if there is to be no possible arbitrage when one is allowed to bet both for and against any of the outcomes?

Exercise 6.4 Consider an experiment with three possible outcomes and odds as follows.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Odds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Suppose that one may choose any pair of outcomes $i \neq j$ and bet that the outcome will be either $i$ or $j$. What should the odds be on these three bets if an arbitrage opportunity is to be avoided?

Exercise 6.5 In Example 6.1a (see the book!), show that if

$$\sum_{i=1}^{m} \frac{1}{1 + o_i} \neq 1$$

then the betting scheme

$$x_i = \frac{(1 + o_i)^{-1}}{1 - \sum_{i=1}^{m}(1 + o_i)^{-1}}, \quad i = 1, \ldots, m,$$

will always yield a gain of exactly 1.