Problem Set # 2

Due date: Monday, May 11th, 2009 at the beginning of the class.

All the problems are taken from the 2nd edition of the book *An Elementary Introduction to Mathematical Finance* by Sheldon M. Ross.

Exercise 6.7 Suppose that, in each period, the cost of a security either goes up by a factor of 2 or goes down by a factor of 1/2 (i.e., \( u = 2, d = 1/2 \)). If the initial price of the security is 100, determine the no-arbitrage cost of a call option to purchase the security at the end of two periods for a price of 150.

Exercise 6.11 The price of a security in each time period is its price in the previous time period multiplied either by \( u = 1.25 \) or by \( d = .8 \). The initial price of the security is 100. Consider the following “exotic” European call option that expires after five periods and has a strike price of 100. What makes this option exotic is that it becomes alive only if the price after two periods is strictly less than 100. That is, it becomes alive only if the price decreases in the first two periods. The final payoff of this option is

\[
\text{payoff at time } 5 = I(S(5) - 100)_+,\n\]

where \( I = 1 \) if \( S(2) < 100 \) and \( I = 0 \) if \( S(2) \geq 100 \). Suppose the interest rate per period is \( r = .1 \).

(a) What is the no-arbitrage cost (at time 0) of this option?

(b) Is the cost of part (a) unique? Briefly explain.

(c) If each price change is equally likely to be an up or a down movement, what is the expected amount that an option holder receives at the time of expiration?

Exercise 2.9 A model for the movement of a stock supposes that, if the present price of the stock is \( s \), then — after one time period — it will either be \( us \) with probability \( p \) or \( ds \) with probability \( 1 - p \). Assuming that successive movements are independent, approximate the probability that the stock’s price will be up at least 30% after the next 1000 time periods if \( u = 1.012, \ d = .990 \), and \( p = .52 \).

Continued on next page
Exercise 7.2 The prices of a certain security follow a geometric Brownian motion with parameters $\mu = .12$ and $\sigma = .24$. If the security’s price is presently 40, what is the probability that a call option, having four months until its expiration time and with a strike price of $K = 42$, will be exercised? (A security whose price at the time of expiration of a call option is above the strike price is said to finish *in the money*.)

Exercise 7.3 If the interest rate is 8%, what is the risk-neutral valuation of the call option specified in Exercise 7.2?

Exercise 7.5 A security’s price follows geometric Brownian motion with drift parameter .06 and volatility parameter .3.

(a) What is the probability that the price of the security in six months is less than 90% of what it is today?

(b) Consider a newly instituted investment that, for an initial cost of $A$, returns you 100 in six months if the price at that time is less than 90% of what it initially was but returns you 0 otherwise. What must be the value of $A$ in order for this investment’s introduction *not* to allow an arbitrage?

Exercise 7.6 The price of a certain security follows a geometric Brownian motion with drift parameter $\mu = .05$ and volatility parameter $\sigma = .3$. The present price of this security is .95.

(a) If the interest rate is 4%, finde the no-arbitrage cost of a call option that expires in three months and has exercise price 100.

(b) What is the probability that the call option of part (a) is worthless at the time of expiration?

(c) Suppose that a new type of investment on the security is being traded. This investment returns 50 at the end of one year if the price six months after purchasing the investment is at least 105 and the price one year after purchase is at least as much as the price was after six months. Determine the no-arbitrage cost of this investment.

Exercise 7.7 A European *asset-or-nothing* call pays its holder a fixed amount $F$ if the price at expiration time is larger than $K$ and pays 0 otherwise. Find the risk-neutral valuation of such a call — one that expires in six month’s time and has $F = 100$ and $K = 40$ — if the present price of the security is 38, its volatility is .32, and interest rate is 6%.

Exercise 7.8 If the drift parameter of the geometric Brownian motion is 0, find the expected payoff of the asset-or-nothing call in Exercise 7.7.