Erwartungswert der Binomialvert., $X \sim \mathrm{Bin}(n,p)$

$$E[X] = \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \sum_{k=1}^{n} k \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

$$= \sum_{k=1}^{n} \frac{n \cdot (n-1)!}{(k-1)!(n-k)!} p^k (1-p)^{n-k}$$

$$= n \sum_{k=1}^{n} \binom{n-1}{k-1} p^k (1-p)^{n-k}$$

$$= n \sum_{k=0}^{n-1} \binom{n-1}{k} p^{k+1} (1-p)^{n-k-1}$$

$$= np \sum_{k=0}^{n-1} \underbrace{\binom{n-1}{k} p^k (1-p)^{n-1-k}}_{=1,\ \text{Wahrsch. für eine } \mathrm{Bin}(n-1,p) \text{ vert.}}$$

Zufallsgröße

$\downarrow$

$= np \qquad \square$

## Linearität von $E$:

$$E[aX+bY] = \sum_{k,l} (ak+bl)\, P[X=k \text{ und } Y=l]$$

$$= a \sum_{k,l} k\, P[\ldots] + b \sum_{k,l} l\, P[\ldots]$$

$$= a\, E[X] + b\, E[Y]$$

# Varianz

$$\mathrm{Var}(X) = E\left[(X - E[X])^2\right]$$

$$= E\left[X^2 - 2XE[X] + (E[X])^2\right]$$

$$\underset{\text{Lin.}}{=} E[X^2] - 2E[X]E[X] + (E[X])^2$$

$$= E[X^2] - (E[X])^2 \qquad \square$$

Erwartungstreue: $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ Mittelwert

$$x_i \longrightarrow X_i$$

$$E[\bar{X}] = \frac{1}{n}\sum_{i=1}^{n} E[X_i] = \frac{1}{n}\sum_{i=1}^{n} E[X] = E[X]$$

$$S_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2 \qquad \text{empirische Varianz}$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \frac{1}{n}\sum_{j=1}^{n} x_j\right)^2$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i^2 - \frac{2}{n}\sum_{j=1}^{n} x_i x_j + \frac{1}{n^2}\left(\sum_{j=1}^{n} x_j\right)^2\right)$$

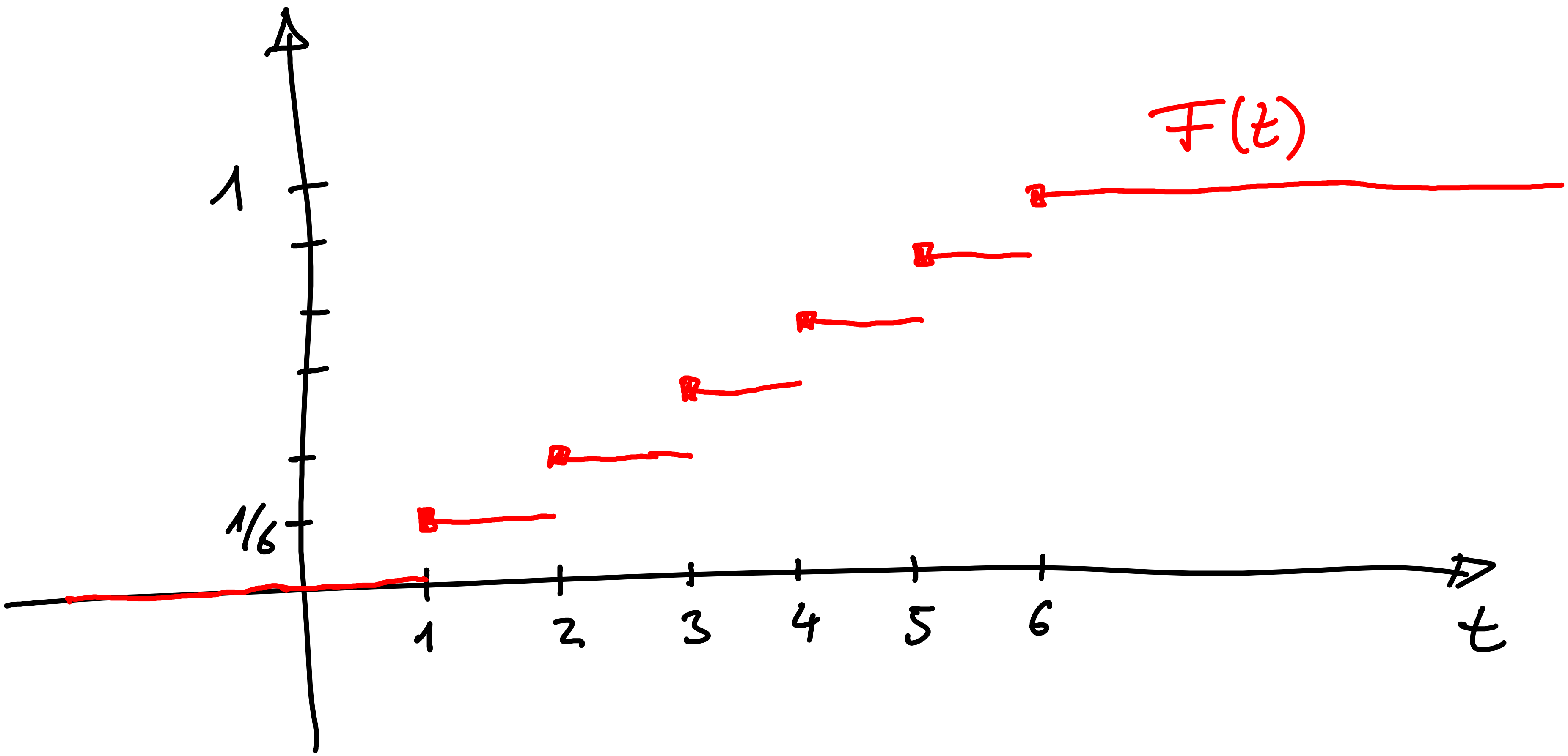
$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 \underbrace{- \frac{2}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j + \frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{n} x_j x_i}_{= -\frac{1}{n}\sum_j \sum_i \dots}\right)$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 \underbrace{\left(1-\frac{1}{n}\right)}_{\frac{n-1}{n}} - \frac{1}{n}\sum_{j=1}^{n}\sum_{i \neq j} x_i x_j\right)$$

$$E[S_x^2] = \frac{1}{n}\sum_{i=1}^{n} \underbrace{E[X_i^2]}_{E[X^2]} - \frac{1}{n(n-1)}\sum_{j=1}^{n}\sum_{i\neq j} \underbrace{E[X_i]E[X_j]}_{(E[X])^2}$$

$$= E[X^2] - (E[X])^2$$

Verteilungsfkt. fairer Würfel ($X$ = Erg. $\in \{1,2,3,4,5,6\}$)



stetige Verteilung