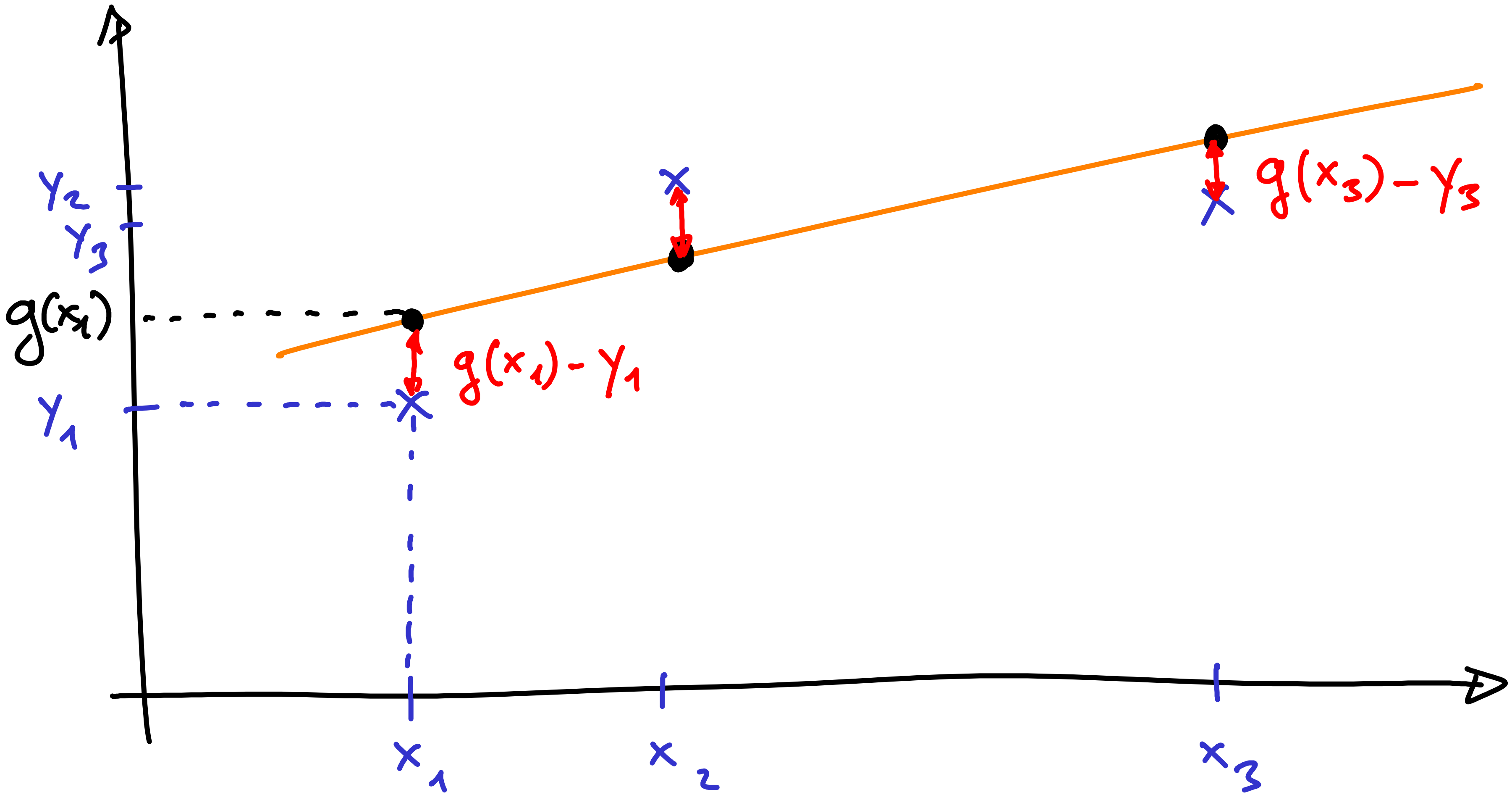
Denken Sie bitte an die

Vorlesungsumfrage

→ Aufforderung kam per Email



$$\underbrace{D}_{D(m,b)} = \sqrt{\sum_{i=1}^{3} \left(g(x_i) - y_i\right)^2}$$

$g(x_i) = m x_i + b$

hängt von m und b ab

$$\sum_{i=1}^{n} 2(mx_i + b - y_i) = 0 \quad \Big| \cdot \frac{1}{2}$$

$$\underbrace{\sum_{i=1}^{n} (mx_i)}_{= m \underline{\sum_{i=1}^{n} x_i}} + \underbrace{\sum_{i=1}^{n} b}_{nb} - \underline{\sum_{i=1}^{n} y_i} = 0 \quad \Big| + \sum_{i=1}^{n} y_i$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \ , \quad \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

$$\underset{\sim}{n}\, m \bar{x} + \underset{\sim}{n}\, b = \underset{\sim}{n}\, \bar{y}$$

$$\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

$$= \sum_{i=1}^{n} \left(x_i y_i - x_i \bar{y} - \bar{x} y_i + \bar{x}\bar{y} \right)$$

$$= \sum_{i=1}^{n} x_i y_i - \bar{y} \underbrace{\sum_{i=1}^{n} x_i}_{n\bar{x}} - \bar{x} \underbrace{\sum_{i=1}^{n} y_i}_{n\bar{y}} + n\bar{x}\bar{y}$$

$$= \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}$$ ← so im LGS

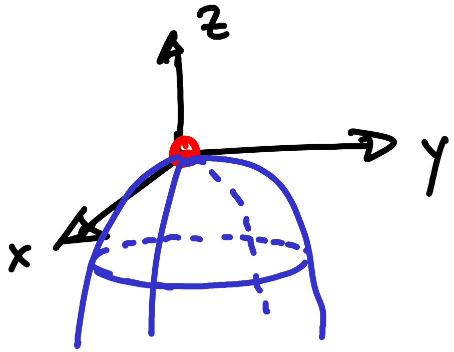
analog $\sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} (x_i - \bar{x})(x_i - \bar{x}) = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$

Durften wir durch

$$\underbrace{\sum_{i=1}^{n} (x_i - \overline{x})^2}_{\geqslant 0 \text{ und } = 0 \text{ nur, falls } x_i = \overline{x}}$$ teilen ?

d.h. wenn nicht alle x_i gleich sind, dann "Ja!"

Definitheit

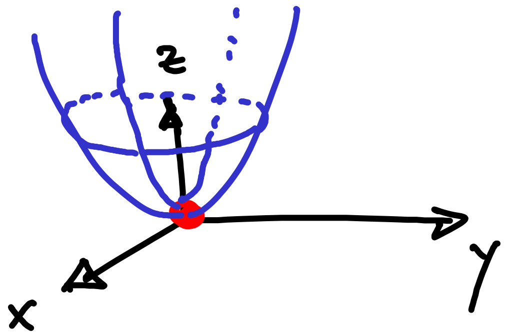


Maximum

z.B. $z = -x^2 - y^2$

$$z'' = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

negativ definit

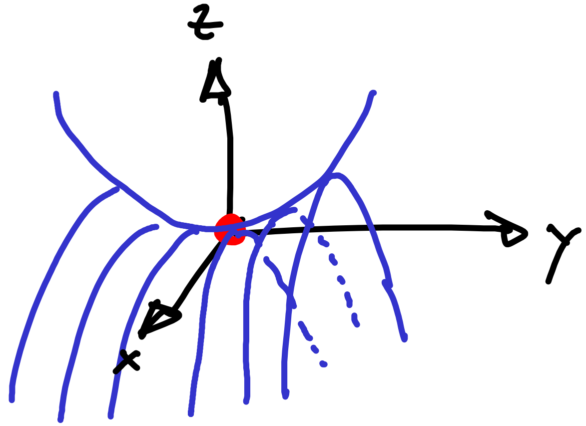


Minimum

z.B. $z = x^2 + y^2$

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

positiv definit



Sattel

z.B. $z = -x^2 + y^2$

$$\begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$$

indefinit

Definitheit von 2×2-Matrizen

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad \vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\vec{x}^T A \vec{x} = (x, y) \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = (x, y) \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

$$= a x^2 + \underbrace{b xy + c yx}_{= 2bxy} + d y^2$$

symmetrische Matrix
$c = b$

quadratische Ergänzung

$$= a \left(x^2 + \frac{2b}{a} xy \right) + d y^2$$

$$= a \left(\left(x + \frac{b}{a} y\right)^2 - \frac{b^2}{a^2} y^2 \right) + d y^2$$

$$= \underline{\underline{a}} \underbrace{\left(x + \frac{b}{a} y\right)^2}_{\geqslant 0} + \underbrace{\left(d - \frac{b^2}{a}\right)}_{\frac{ad - b^2}{a}} \underbrace{y^2}_{\geqslant 0}$$

Falls $a > 0$ und $ad - b^2 > 0$ dann ist A positiv definit

übrigens

$$\begin{pmatrix} a & b \\ b & d \end{pmatrix} \rightsquigarrow ad - b^2$$

2. Ableitung bei Regressionsanalyse

$$\begin{pmatrix} 2n & 2n\bar{x} \\ 2n\bar{x} & 2\sum_{i=1}^{n} x_i^2 \end{pmatrix}$$

① $2n > 0$

② $4n \sum_{i=1}^{n} x_i^2 - 4n^2\bar{x}^2$

$= 4n \left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2 \right)$ ← kennen wir von oben

> 0 solange nicht alle x_i gleich sind

$\Rightarrow f''$ pos. def. $\Rightarrow$ Minimum