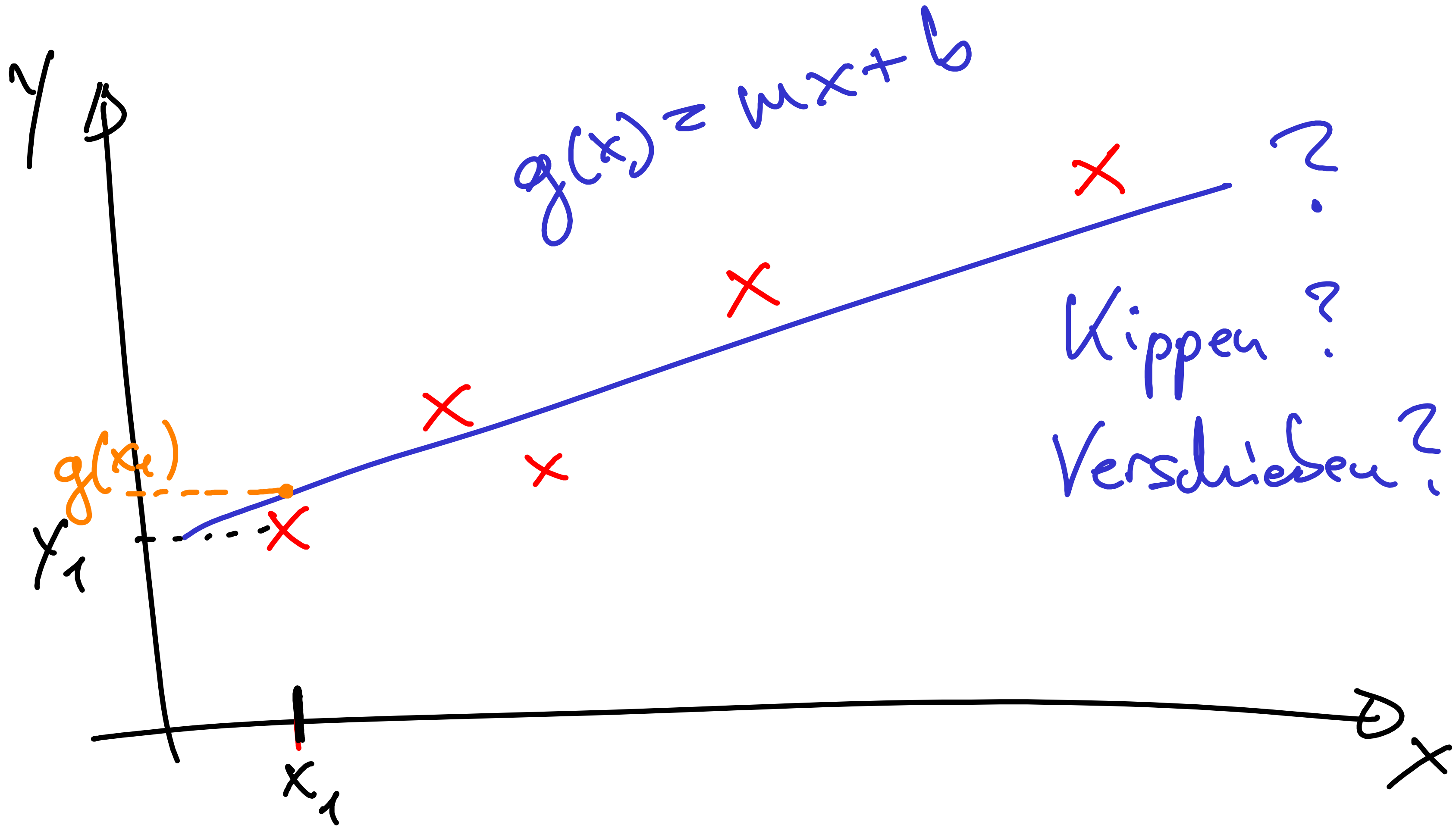


$g(x) = mx + b$
?
Kippen ?
Verschieben ?
$g(x_1)$
y_1
x_1
y
x

(1) $f(x,y) = x^2 + 2y^2 > 0 \quad \forall (x,y) \neq (0,0)$

$$H = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix} \quad \text{pos. definit}$$

$$f(x,y) = (x,y)\, H \begin{pmatrix} x \\ y \end{pmatrix}$$

(2) $f(x,y) = -x^2 - 2y^2 < 0 \quad \forall (x,y) \neq (0,0)$

$$H = \begin{pmatrix} -2 & 0 \\ 0 & -4 \end{pmatrix} \quad \text{neg. definit}$$

(3) $f(x,y) = x^2 - 2y^2$

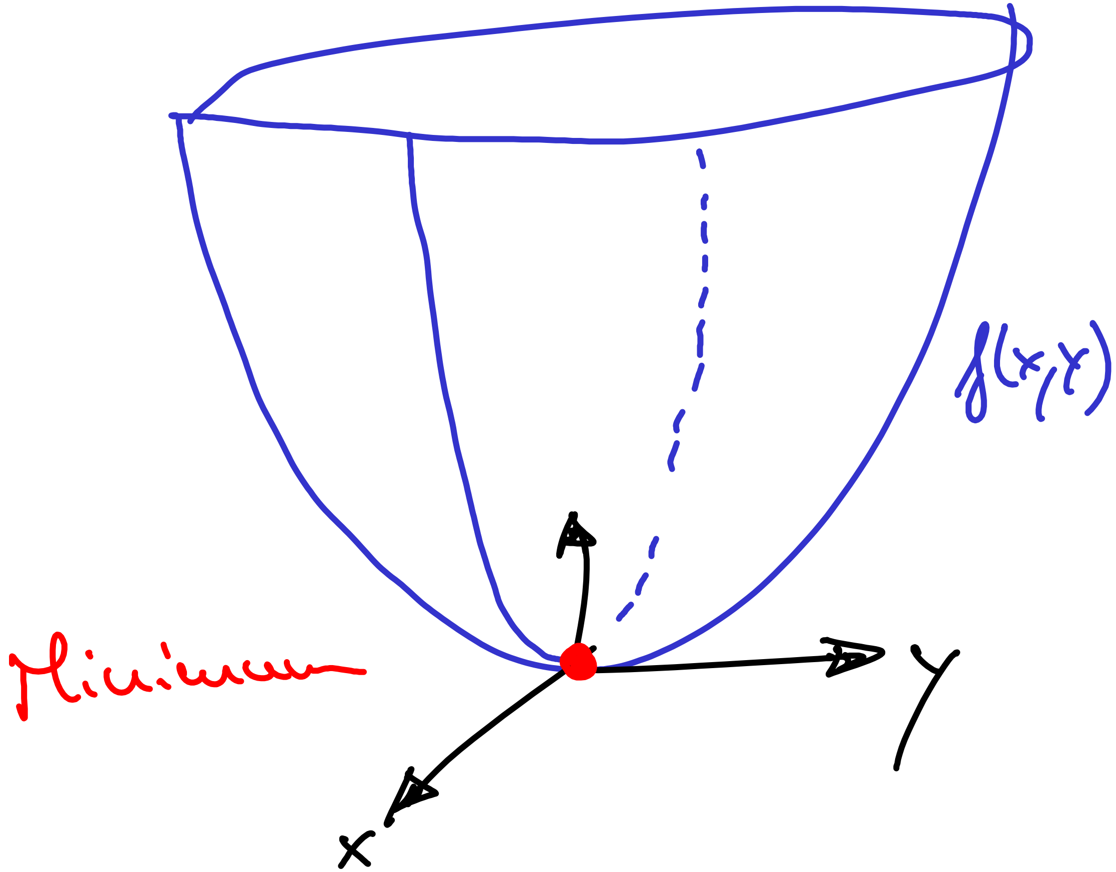
$$H = \begin{pmatrix} 2 & 0 \\ 0 & -4 \end{pmatrix} \quad \text{indefinit}$$

z.B. $f(1,0) = 1$

$f(0,1) = -2$

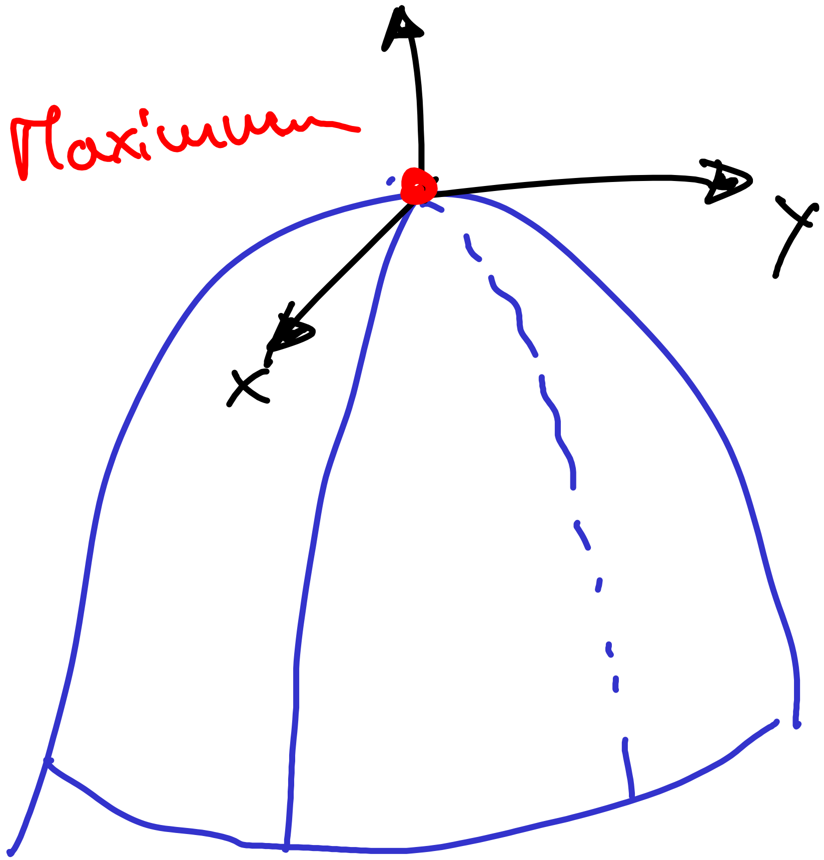
(1)

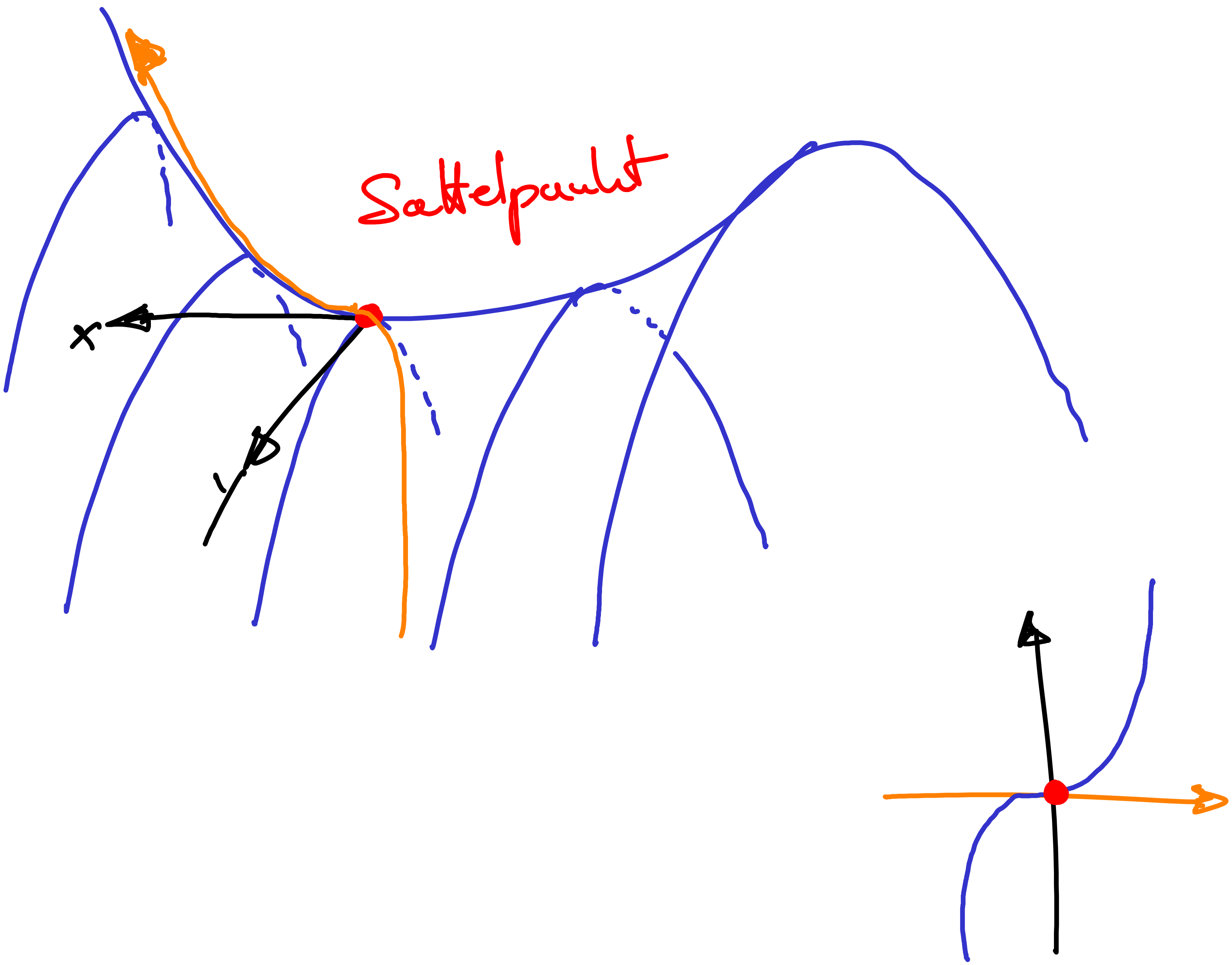
H pos. definit



(2)

H neg. definit




Sattelpunkt
x
y

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\sum_{i=1}^{n} (\bar{x} - x_i)^2 > 0$$ da nicht alle x_i gleich

$$= \sum_{i=1}^{n} \left(\bar{x}^2 - 2\bar{x}x_i + x_i^2\right)$$

$$= \sum_{i=1}^{n} \left(\left(\sum_{j=1}^{n} x_j\right)^2 \frac{1}{n^2} - \frac{2x_i}{n}\sum_{j=1}^{n} x_j + x_i^2 \right)$$

$$= \frac{n}{n^2}\underbrace{\left(\sum_{j=1}^{n} x_j\right)^2} - \frac{2}{n}\underbrace{\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{j=1}^{n} x_j\right)} + \sum_{i=1}^{n} x_i^2$$

$$=$$

$$= -\frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2 + \sum_{i=1}^{n} x_i^2 > 0 \quad \Bigg| \; \frac{n}{\sum_{i=1}^{n} x_i^2}$$

$$-\frac{\left(\sum_{i=1}^{n} x_i\right)^2}{\sum_{i=1}^{n} x_i^2} + n > 0 \qquad \square$$

Anhang: Diskussionen nach der Vorlesung

$$f(x,y) = x^2 - 2y^2$$

$$\frac{\partial f}{\partial x} = 2x \quad , \quad \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = 0$$

$$f(x,y) = x^2 y^3$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(3x^2y^2\right) = 6xy^2$$

$$(\alpha, \beta)\begin{pmatrix}4 & 1\\ 1 & 3\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

$$= (\alpha,\beta)\begin{pmatrix}4\alpha+\beta\\ \alpha+3\beta\end{pmatrix} = (4\alpha^2+\alpha\beta)+(\beta\alpha+3\beta^2)$$

$$4x^2 + 10xy - 3y^2 \qquad 10/2$$

$$= (x,y)\begin{pmatrix}4 & 8\\ 5 & -3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$$