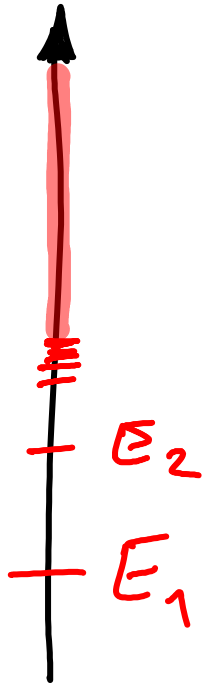
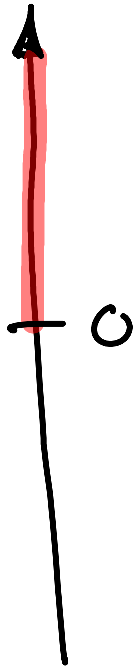
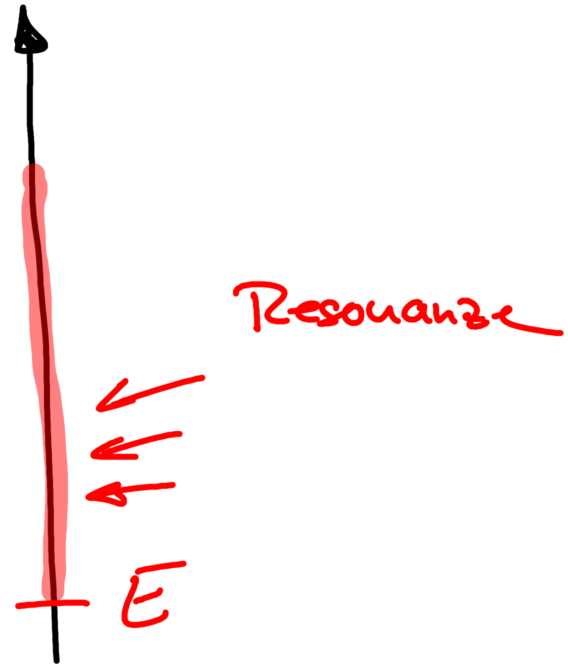


H_{e^-}

(z.B. H-Atom)



H_f



$H_{e^-} + H_f + H_{WW}$

physikalische Ham.-Op. (1 e^-)

$$H = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla + \frac{e}{c}\vec{A}(\vec{x})\right)^2 + c\hbar \sum_{\lambda=1,2} \int_{\mathbb{R}^3} |\vec{k}|\, a^+_\lambda(\vec{k})\, a_\lambda(\vec{k})\, d^3k$$

$$- \sum_n \frac{Z_n e^2}{|\vec{x} - \vec{R}_n|}$$

Summe über Kerne

Position d. Kerne

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$ Feinstrukturkonstante

$$e = (\hbar c \alpha)^{1/2}$$

$$\vec{x} \longmapsto \alpha \vec{x} \qquad (\vec{x} = \alpha \vec{y})$$

$$H = \frac{1}{2m}\left(\frac{\hbar}{i\alpha}\nabla_y + \frac{e}{c}\hat{A}(\alpha\vec{y})\right)^2 + c\hbar \sum_\lambda \int \ldots dk - \frac{e^2}{\alpha} \sum_n \frac{Z_n}{|\vec{y} - \frac{R_n}{\alpha}|}$$

$\hbar = c = 1 : \quad \alpha = e^2$

$$H = \frac{1}{2m\alpha^2}\left(\frac{1}{i}\nabla_y + \alpha^{3/2}\hat{A}(\alpha\vec{y})\right)^2 + \sum_\lambda \int \ldots dh - \sum_n \frac{Z_n}{|\vec{y} - \tilde{\vec{R}}_n|}$$

① Kernpositionen $\tilde{\vec{R}}_n$ sind α-abhängig

② $m = \dfrac{1}{2\alpha^2}$, $\alpha \longrightarrow 0$

$$\left(\frac{e}{m} = \frac{\alpha^2}{1/2\alpha^2} = 2\alpha^4 \longrightarrow 0\right)$$

$$\frac{1}{x-x_0 \pm i\varepsilon} = P\,\frac{1}{x-x_0} \mp i\pi\,\delta(x-x_0)$$

zu Satz (Krit. f. a.c. Spektrum)

$$\frac{1}{2}\left\langle \varphi, \left[E_{[a',b']} + E_{(a',b')}\right]\varphi\right\rangle$$

$$= \lim_{\varepsilon\to 0+} \frac{1}{\pi}\int_{a'}^{b'} \langle \varphi, \operatorname{Im} R(\mu+i\varepsilon)\varphi\rangle\, d\mu$$

(Stone Formel)

$E_{(a',b')} \leq E_{[a',b']}$, für $(a',b') \subseteq (a,b)$

$$\langle \varphi, E_{(a',b')}\varphi\rangle \leq \frac{1}{\pi}\int_{a'}^{b'} C(\varphi)\, d\mu = \frac{C(\varphi)}{\pi}|b'-a'|$$

(also $\to 0$ für $a' \to b'$)

$$\langle A \rangle = \sqrt{1 + A^2}$$

Virial Theorem: Annahme Eigenflt. ψ
(dort wo's interessiert)

$$\langle \psi, [H, iA]\, \psi \rangle = 0 \quad \forall \text{ zur Abschätzung}$$

$$[H, iA] = C^\dagger C\,, \quad \ker C = \{0\} \qquad (*)$$

donut $\quad R(z) := (H - z)^{-1}$

$$\| C R(\mu \pm i\varepsilon) \|^2$$

$$= \| R(\mu \mp i\varepsilon) C^\dagger C R(\mu \pm i\varepsilon) \|$$

$$\overset{(*)}{=} \| R(\mu \mp i\varepsilon) [H - \mu \mp i\varepsilon, iA] R(\mu \pm i\varepsilon) \|$$

$$\leq \| R(\mu \mp i\varepsilon) A \| + \| R(\mu \mp i\varepsilon) (H - \underbrace{\mu \mp i\varepsilon}_{\text{“}\pm 2i\varepsilon\text{”}}) A R(\mu \pm i\varepsilon) \|$$

$$\leq \| R(\mu \mp i\varepsilon) A \| + \| A R(\mu \pm i\varepsilon) \| + 2\varepsilon \| R(\dots) A R(\dots) \|$$

$$\leq \frac{4}{\varepsilon} \| A \|$$

Weiter

$$2\,\| C \,\mathrm{Im}\, R(\mu+i\varepsilon)\, C^+ \|$$

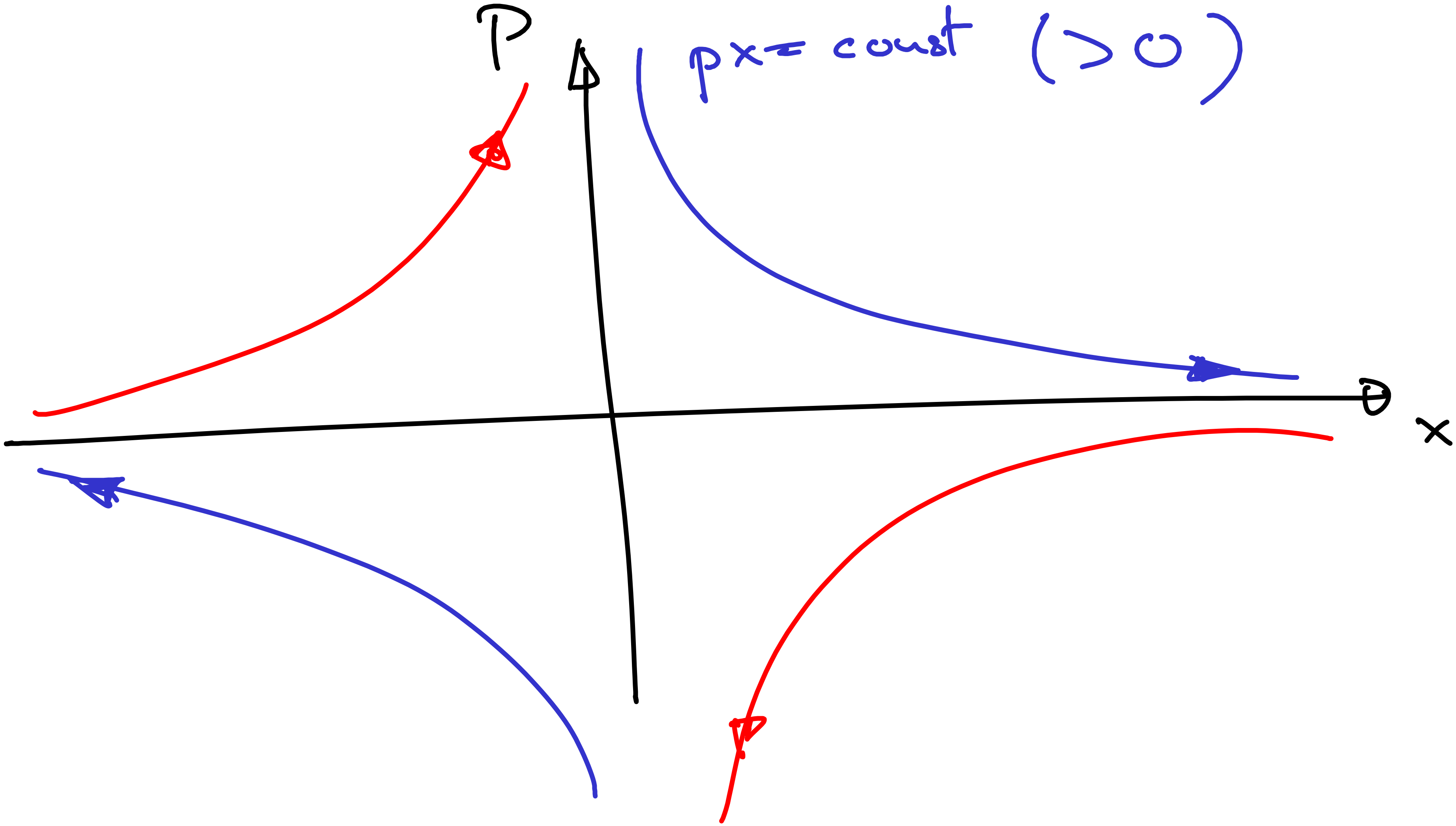
$$= \| C\, R(\mu+i\varepsilon)\, 2i\varepsilon\, R(\mu-i\varepsilon)\, C^+ \|$$

$$\leq 8\, \| A \| \qquad \square$$

$$2\varepsilon \| C\, R(\mu+i\varepsilon) \|^2$$

$$e^{\varepsilon A} H e^{-\varepsilon A} = H + \varepsilon [A, H] + \ldots$$

$$= H + i\varepsilon [H, iA] + \ldots$$



p
px = const (> 0)
x