

Bewegungsgl. : $m \ddot{x} = -\nabla V(x)$

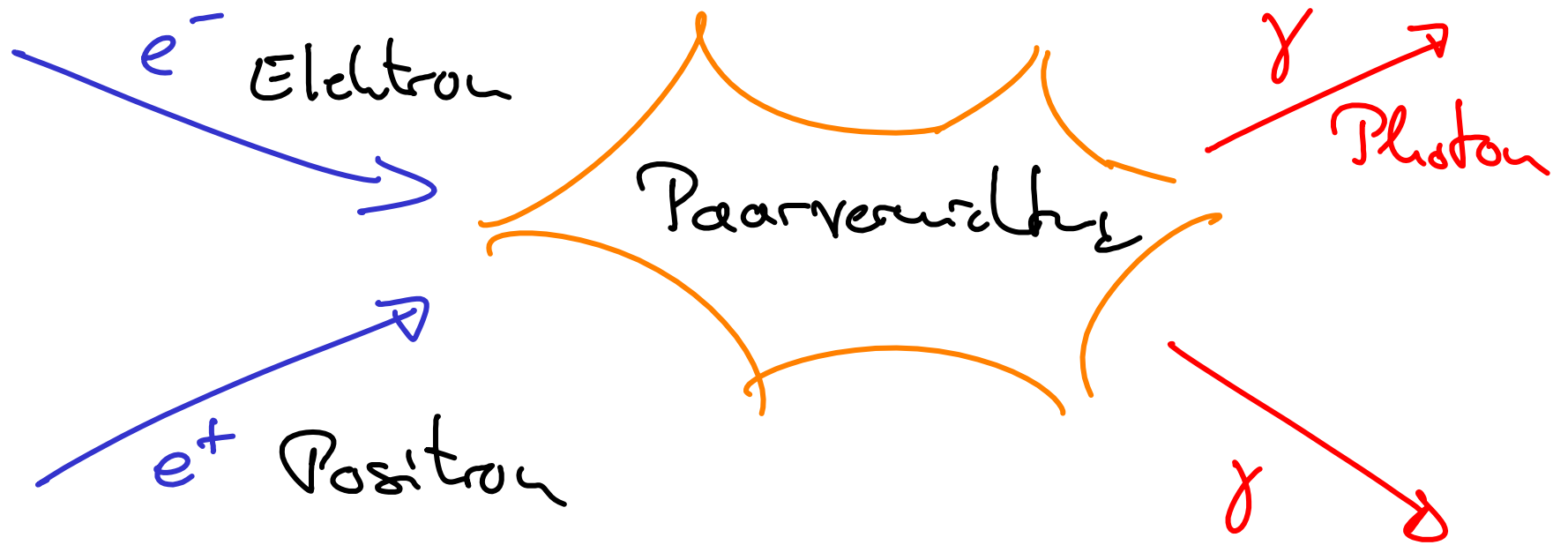
Kommutator : $[A, B] = AB - BA$

Hamilton fkt. : $H(p, x) = \frac{p^2}{2m} + V(x)$

"quantisieren"

$$p^2 = -\hbar^2 \Delta$$

Hamilton op. : $\hat{H} = -\frac{\hbar^2}{2m} \Delta + V(x)$



$$\bullet \left(a^\dagger(\psi) \Phi \right)_0 = 0$$

$$\bullet \left(a^\dagger(\psi) \Omega \right)_1^{(x_1)} = \frac{1}{\sqrt{1}} \sum_{j=1}^1 \psi(x_j) \cdot 1 = \psi(x_1)$$

alle anderen Null

$$\bullet \left(a(\underline{\psi}) (0, \underline{\psi}, 0, \dots) \right)_0 = \sqrt{1} \int \underline{\psi}(x) \underline{\psi}(x) d^3x$$

$$= \|\psi\|^2 \quad (= \alpha)$$

$$\bullet \left(a^\dagger(\psi) (0, \phi, 0, \dots) \right)_2^{(x_1, x_2)} = \frac{1}{\sqrt{2}} \left(\psi(x_1) \phi(x_2) + \psi(x_2) \phi(x_1) \right)$$

$$\left(\underbrace{\sum_j a^+(\varphi_j) \overline{\varphi_j(x)}}_{= a^+(x)} \Omega \right)_1(x_1)$$

$$= \sum_j \underbrace{(a^+(\varphi_j) \Omega)_1(x_1)}_{= \varphi_j(x_1)} \overline{\varphi_j(x)}$$

$$= \delta(x - x_1)$$

$$a(x) = \sum_j a(\varphi_j) \varphi_j(x)$$

$$\sum_j \varphi_j(x) \overline{\varphi_j(x')} = \delta(x-x') \quad \text{Warum?}$$

Vollständigkeit

Integralkerne für Fourierentwicklung in $\{\varphi_j\}$

$$\phi \in L^2(\mathbb{R}^3)$$

$$\phi(x) = \sum_j c_j \varphi_j(x)$$

$$c_j = \langle \varphi_j, \phi \rangle = \int_{\mathbb{R}^3} \overline{\varphi_j(x')} \phi(x') d^3 x'$$

$$= \int_{\mathbb{R}^3} \sum_j \underbrace{\varphi_j(x) \overline{\varphi_j(x')}} \phi(x') d^3 x'$$

Integralkerne d. Entwicklung

$$a(x) = \sum_j a(\psi_j) \psi_j(x)$$

$$\int_{\mathbb{R}^3} \overline{\psi}(x) a(x) d^3x = \sum_j \langle \psi, \psi_j \rangle a(\psi_j)$$
$$= a\left(\sum_j \langle \psi_j, \psi \rangle \psi_j\right) = a(\psi)$$