Group Representations in Physics

Homework Assignment 12 (due on 31 Jan 2018)

Problem 50

Let Θ_λ be the Young diagram with row lenghts $\lambda_i \geq 0$, $\lambda = (\lambda_1, \ldots, \lambda_N)$, and let $\Theta_{\lambda'}$ be given by $\lambda' = (\lambda_1+1, \ldots, \lambda_N+1)$. Show that the SU($N$)-irreps Γ^λ and $\Gamma^{\lambda'}$ are equivalent.

HINT: Use the graphical rule from Section 7.4 and the result of Problem 48.

Problem 51

Let Γ^λ be an SU(3)-irrep with Young diagram Θ_λ. Determine how often Γ^λ appears in the product rep defined by $\Theta_\lambda \otimes \boxminus$.

HINT: Study separately the cases of rectangular Young diagrams Θ_λ (with one or two rows) and of non-rectangular diagrams.

Problem 52

Decompose the product rep $\square \otimes \square \otimes \square$ of SU(3) into irreducible representations. Use the notation of Problem 33 (e.g. $|uds\rangle = |u\rangle \otimes |d\rangle \otimes |s\rangle \in \square^{\otimes 3}$) and explicitly construct bases for the irreducible invariant subspaces. Compare with the results of Problem 33. What is the relation between the irreducible subspaces with respect to SU(3) and those with respect to S_3?