```
> restart;
```

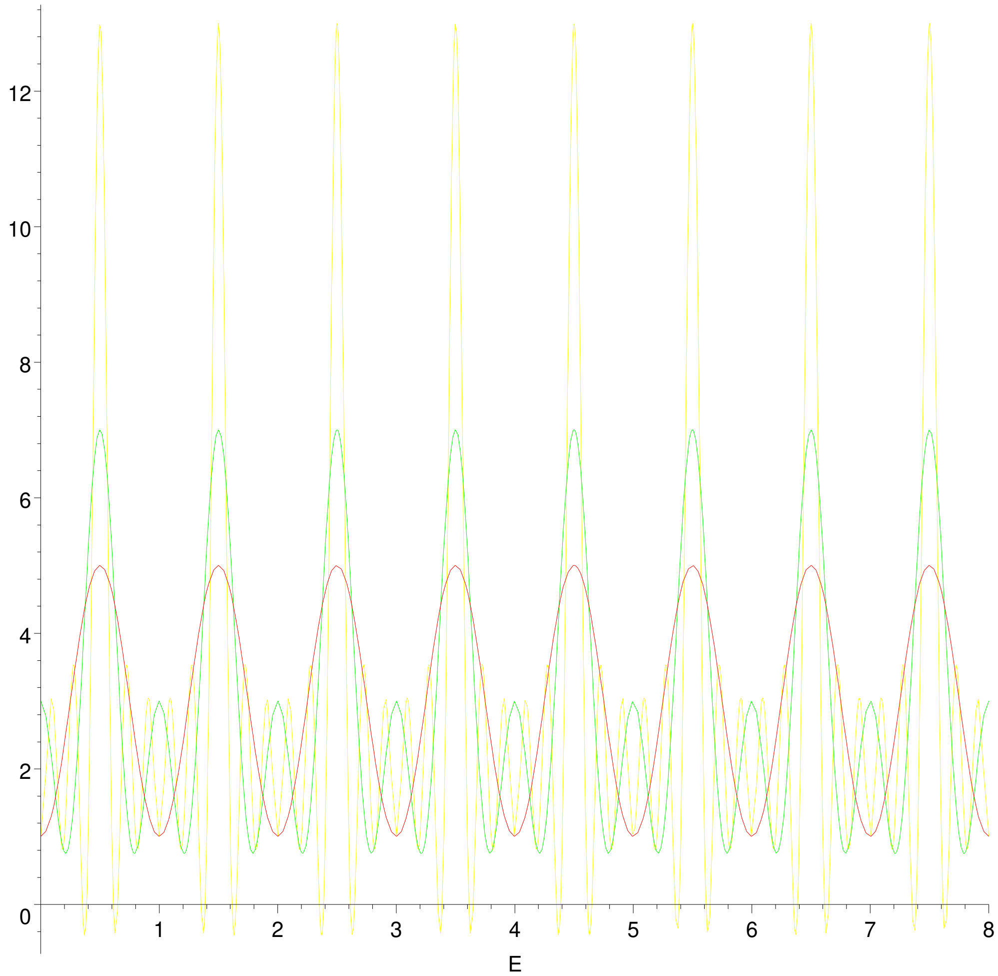
The trace formula for the density of states of the harmonic oscillator with $\frac{h\,\omega}{2\,\pi} = 1$ is given by the limit $K \to \infty$ of the expression

$$d(E, K) := 1 + \left(\sum_{k=0}^{K} 2\cos(2\,\pi\,E\,k + k\,\pi)\right)$$

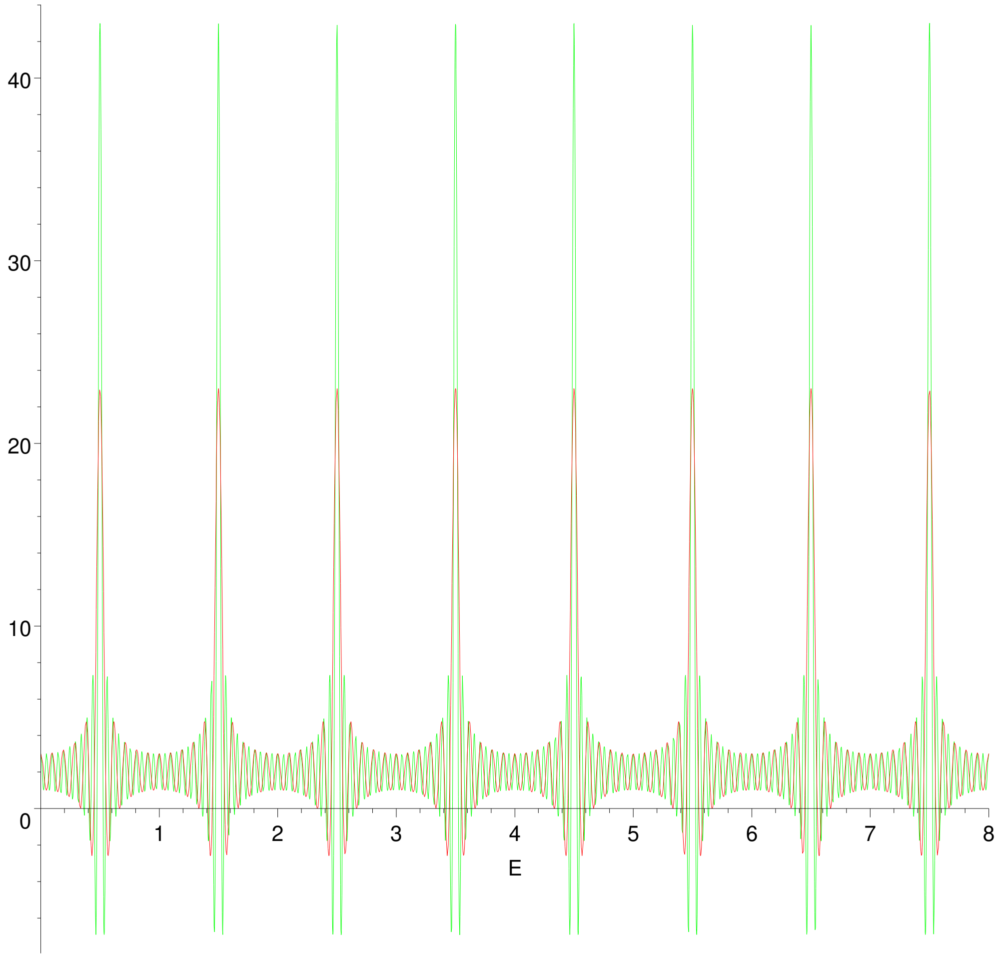
$$\begin{aligned} d(E, K) := {} & 1 - 2\cos(\pi\,E\,(K+1))^2\cos((K+1)\,\pi) + \frac{\sin(\pi\,E)\sin((K+1)\,\pi)}{\cos(\pi\,E)} \\ & + (-\cos(\pi\,E)^2 + 2 - \sin(\pi\,E)^2)\cos((K+1)\,\pi) \\ & - 2\,\frac{\sin(\pi\,E)\sin(\pi\,E\,(K+1))\cos(\pi\,E\,(K+1))\cos((K+1)\,\pi)}{\cos(\pi\,E)} \\ & + 2\sin(\pi\,E\,(K+1))\cos(\pi\,E\,(K+1))\sin((K+1)\,\pi) \\ & - 2\,\frac{\sin(\pi\,E)\cos(\pi\,E\,(K+1))^2\sin((K+1)\,\pi)}{\cos(\pi\,E)} + \cos(\pi\,E)^2 + \sin(\pi\,E)^2 \end{aligned}$$

When plotting this expression using different cut-offs $K$ ...

```
> plot([subs(K=1,d(E,K)), subs(K=2,d(E,K)),
  subs(K=5,d(E,K))],E=0..8);
```




```
> plot([subs(K=10,d(E,K)), subs(K=20,d(E,K))],E=0..8,
  numpoints=500);
```




...we observe peaks at the positions $n+\frac{1}{2}$ which are the eigen-energies of the harmonic oscillator.

In order to also make a quantative statement about the convergence of the trace formula we evaluate the spectral density on a normalised Gaussian with width ε and get the smooth trace formula

$$\sum_{n=0}^{\infty}\frac{\mathbf{e}^{\left(-\frac{(E-E_n)^2}{2\,\varepsilon^2}\right)}}{\sqrt{2\,\pi\,\varepsilon^2}}=1+\left(\sum_{k=1}^{\infty}(-1)^k\cos(2\,\pi\,k\,E)\,\mathbf{e}^{(-2\,\pi^2\,\varepsilon^2\,k^2)}\right)$$

where the quantum expression ($N\rightarrow\infty$)

$$d_{qm}(E,\varepsilon,N):=\sum_{n=0}^{N}\frac{\mathbf{e}^{\left(-\frac{\left(E-n+\frac{1}{2}\right)^2}{2\,\varepsilon^2}\right)}}{\sqrt{2\,\pi\,\varepsilon^2}}$$

has Gaussian maxima of width ε at the positions of the eigen-energies $E_n = n + \frac{1}{2}$ ; in the the classical expression

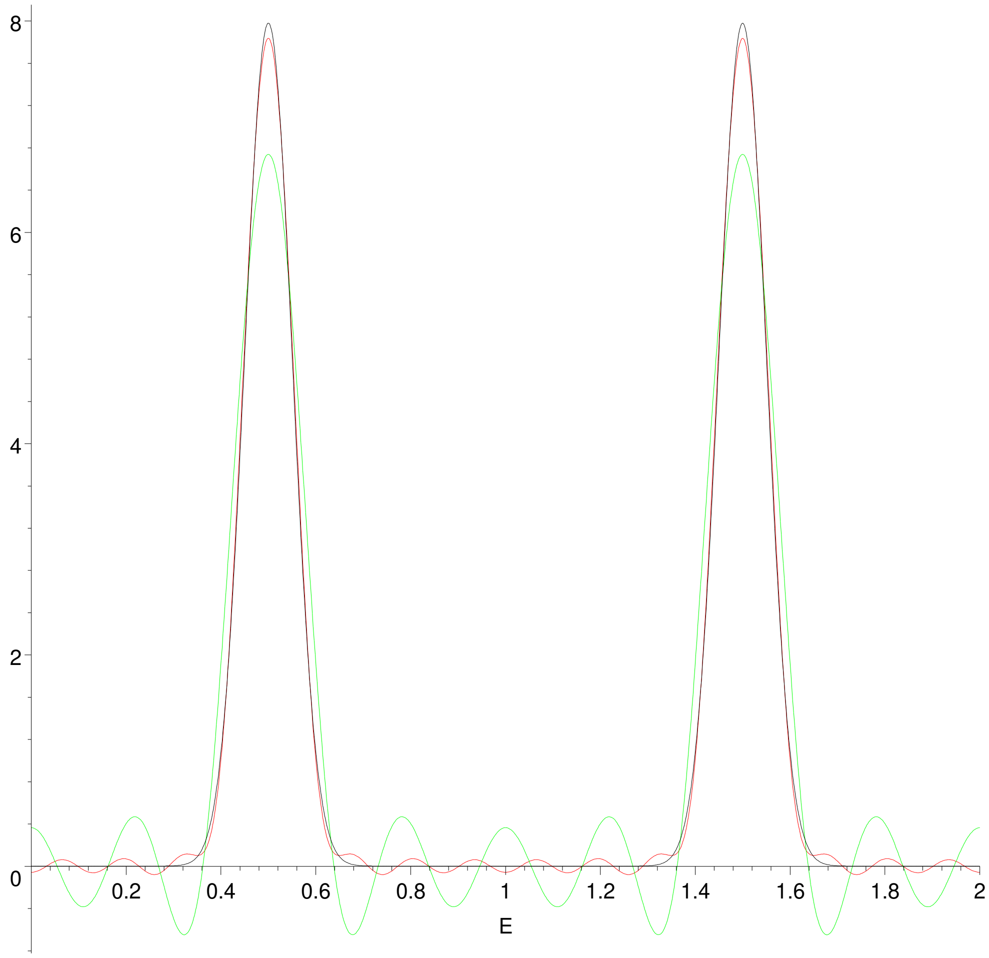
$$d_{cl}(E, \varepsilon, K) := 1 + \left( \sum_{k=1}^{K} 2\,(-1)^k \cos(2\,\pi\,k\,E)\,\mathbf{e}^{(-2\,\pi^2\,\varepsilon^2\,k^2)} \right)$$

the contributions of long orbits (large $k$) are damped.

$$d_{qm}(E, \varepsilon, N) := \sum_{n=0}^{N} \left( \frac{1}{2} \frac{\mathbf{e}^{\left(-1/2\frac{(E-n+1/2)^2}{\varepsilon^2}\right)}\sqrt{2}}{\sqrt{\pi\,\varepsilon^2}} \right)$$

$$d_{cl}(E, \varepsilon, K) := 1 + \left( \sum_{k=1}^{K} (2\,(-1)^k \cos(2\,\pi\,E\,k)\,\mathbf{e}^{(-2\,\pi^2\,\varepsilon^2\,k^2)}) \right)$$

```
> plot([subs(N=10, subs(epsilon=0.05, d[qm](E,epsilon,N))),
        subs(K= 4, subs(epsilon=0.05, d[cl](E,epsilon,K))),
        subs(K= 7, subs(epsilon=0.05, d[cl](E,epsilon,K)))],
  E=0..2, numpoints=1000, color=[black,green,red]);
```




When comparing the quantum mechanical expression (black) with the classical expression for different cut-offs $K$ (green: 4, red: 7) we also observe quantitative agreement.