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Theory of Quantum Chaos

In problem 3 we discussed the standard map on the torus $\mathbb{R}^2/(2\pi\mathbb{Z})^2$.

In problem 12 we discussed general kinematic aspects of quantised maps of the torus $\mathbb{R}^2/\mathbb{Z}^2$. We found that $\hbar$ and the finite dimension N of the quantum Hilbert space are related by the condition $2\pi\hbar N = 1$ and that wave functions are supported at points j/N with $j = 0, \ldots, N-1$.

For consistency, we should instead have discussed quantised maps of the torus $\mathbb{R}^2/(2\pi\mathbb{Z})^2$. This can be done in exactly the same way and one finds that

$$\hbar = \frac{2\pi}{N}$$

and that wave functions are supported at points

$$\frac{2\pi j}{N} \quad \text{with} \quad j = 0, \ldots, N-1\,.$$

Finally, the propagator U of the quantum kicked rotator, which is an $N \times N$ unitary matrix, has matrix elements

$$u_{jl} = \frac{1}{\sqrt{\mathrm{i}N}} \, \exp\left[\mathrm{i}\frac{N}{2\pi}\left(\frac{1}{2}\left(2\pi\frac{j-l}{N}\right)^2 - k\cos\left(\frac{2\pi j}{N}\right)\right)\right]\,.$$

Note that we may omit the ‘i’ under the square root, since it is only a global phase, and thus does not change the spectrum.