Eberhard Karls Universität Tübingen Sommersemester 2019
Fachbereich Mathematik
PD Dr. Jörg Zintl

Algebraische Kurven

Übungsaufgaben zum 5. Tutorium am 05.06.2019

Aufgabe 18.
Let $C \subset \mathbb{P}^2$ be a smooth projective plane curve of degree d. Show that there exists a morphism $\varphi : C \to \mathbb{P}^1$ of degree $\deg(\varphi) = d$.

Is it possible to construct examples of morphisms $\varphi : C \to \mathbb{P}^1$ such that $\deg(\varphi) < \deg(C)$?

Aufgabe 19.
Let $C \subset \mathbb{P}^2$ be a smooth projective plane curve of degree $d = 2$. Prove or disprove the following claims about linear equivalence:

a) For any point $P \in C$, there exists a point $Q \in C$, such that $P + Q \sim 0$.

b) For all points $P, Q \in C$ holds $P \sim Q$.

Aufgabe 20. (Keine Abgabe, Präsenzübung)
Let $C \subset \mathbb{P}^2$ be a smooth projective plane curve of degree $d = 3$.

a) Let $P, Q, R, S \in C$. Give a necessary and sufficient condition for the existence of a linear equivalence $P + Q \sim R + S$ of divisors.

b) Let $P, Q, S \in C$. Show that there exists a unique point $R \in C$ such that $P+Q \sim R+S$.

c) Let a point $S \in C$ be fixed. Suppose that S is an inflection point, i.e. there exists a unique tangent L such that $i_S(L, C) = 3$. For two points $P, Q \in C$ define $P \oplus Q := R$ if and only if $P + Q \sim R + S$. Verify that "$\oplus$" defines an Abelian group structure on C.

(Note: On cubic curves holds for any points $T, T' \in C$ the equivalence $T \sim T' \Leftrightarrow T = T'$.)

Abgabe der Lösungen zu Aufgaben 18 und 19 am 05.06.2019 in der Übung.